

$$x^3 - 4 > 12x - 20 \rightarrow x^3 - 12x + 16 > 0 \rightarrow (x-2)(x-2)(x+4) > 0$$

$\frac{x}{2} \quad + \quad -4 \quad - \quad 2$
 $\downarrow$
 بازه‌ی ۱
 $\hookrightarrow (-\infty, -4)$

$$f^{-1}(2) = 4 \rightarrow f(4) = \frac{3(4)}{12} + k = 2 \rightarrow \boxed{k = -10}$$

$$f(7) = 21 - 10 = 11$$

$$f(f(x)) = 3(3x - 10) - 10 = 9x - 30 - 10 = \boxed{9x - 40}$$

$$y = \frac{ax}{x-1} \rightarrow yx - y = ax \rightarrow x(y-a) = y \rightarrow x = \frac{y}{y-a} \Rightarrow f^{-1}(x) = \frac{x}{x-a}$$

$$f^{-1}(2a) = \frac{2a}{a} = a \Rightarrow \boxed{a = 2}$$

$$الف) f \circ f^{-1} = \{(1,1), (2,2), (3,3), (4,4)\}$$

$$ب) \{(3,3), (4,4), (7,7), (9,9)\}$$

$$ج) = \{(2,4), (4,2)\}$$

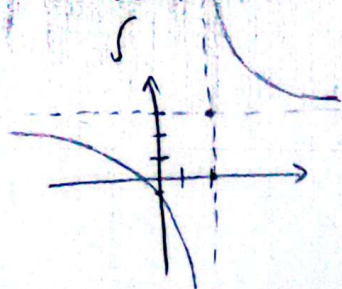
$$د) \{(3,9), (7,3), (9,1)\}$$

$$g^{-1} = \{(1,2), (3,4), (4,6)\} \Rightarrow f \circ g^{-1} = \{(1,4), (3,8), (4,6)\}$$

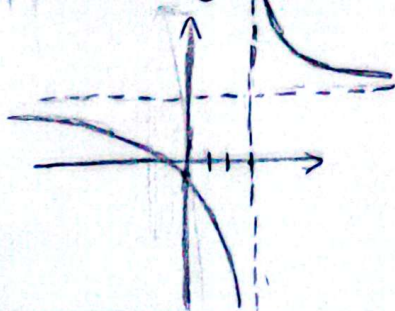
$$h = \{(1,2), (3,4), (4,1), (4,1)\}$$

$$\rightarrow \frac{h}{f \circ g^{-1}} = \{(1, \frac{1}{4}), (3, \frac{1}{4})\}$$

$$y = \frac{3x+1}{x-2}$$

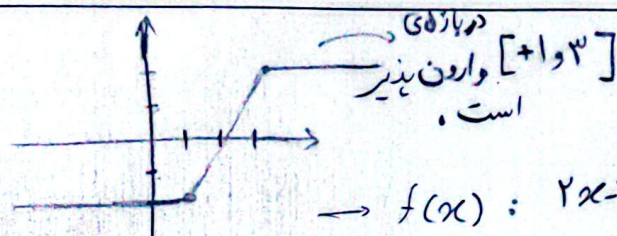


$$\rightarrow yx - 2y = 3x + 1 \rightarrow x(y-3) = 2y+1$$



$$\rightarrow x = \frac{2y+1}{y-3}$$

$$\rightarrow f^{-1}(x) = \frac{2x+1}{x-3}$$



$$\rightarrow f(x) : 2x-4 \text{ و } [1, 3]$$

$$y = 2x-4 \rightarrow x = \frac{y+4}{2} \Rightarrow f^{-1}(x) = \frac{x+4}{2} \begin{cases} R_f^{-1} = [1, 3] \\ D_f^{-1} = [-2, 2] \end{cases}$$

$$f(x) = \begin{cases} x^3+4 & ; x \geq 1 \\ 4x-1 & ; x \leq 0 \end{cases} \begin{cases} R_f = [4, +\infty) \\ R_f = (-\infty, -1] \end{cases}$$

$$1) y = x^3+4 \rightarrow y-4 = x^3 \rightarrow x = \sqrt[3]{y-4} \Rightarrow f^{-1}(x) = \begin{cases} \sqrt[3]{x-4} & ; x \geq 4 \\ \frac{x-1}{4} & ; x \leq -1 \end{cases}$$

$$2) y = 4x-1 \rightarrow x = \frac{y+1}{4} \Rightarrow f^{-1}(x) = \frac{x+1}{4}$$

$$f(x) = x^2 - \frac{(x+1)^3}{x+3}$$

$$f(x) = \frac{x^3 + 3x^2 - x^3 - 3x^2 - 3x - 1}{x+3} = \frac{-3x-1}{x+3}$$

$$f^{-1}(-2) = \frac{12-2}{-4+3} = \frac{10}{-1} = -10$$

$$\rightarrow f : y = \frac{-3x-1}{x+3} \rightarrow xy + 3y = -3x-1$$

$$\Rightarrow x = \frac{-3y-1}{y+3} \rightarrow f^{-1} : y = \frac{-3x-1}{x+3} \Rightarrow \boxed{b=-2}$$

$$y_1 = y_2 \Rightarrow \frac{x_1}{x_1^2+1} = \frac{x_2}{x_2^2+1} \Rightarrow x_1 x_2^2 + x_1 = x_2 x_1^2 + x_2 \Rightarrow x_1 x_2^2 + x_1 - x_2 x_1^2 - x_2 = 0$$

$$\Rightarrow x_1 x_2 (x_2 - x_1) - (x_2 - x_1) = 0 \Rightarrow (x_2 - x_1)(x_1 x_2 - 1) = 0 \begin{cases} x_1 = x_2 \\ x_1 x_2 = 1 \end{cases}$$

تابع f یک به یک نیست پس معکوس پذیر نیست.

$$f \text{ یک به یک نیست} \Rightarrow \left(\frac{1}{3} \text{ و } \frac{2}{3}\right) \text{ و } \left(2 \text{ و } \frac{4}{3}\right) \in f \text{ مثال نقض}$$