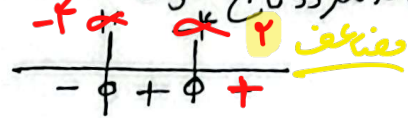


نام و نام خانوادگی ..... شماره ..... کلاس ..... پاسخنامه تشریحی تکلیف شماره ...

از آنجا که هر دو تابع صعودی هستند داریم:

$$x^3 - 4 \geq 12x - 20 \Rightarrow x^3 - 12x + 16 \geq 0$$



$$\Rightarrow \cancel{4 \leq x} \leq 2$$

$$[a \geq -4]$$

$$D_f = R_{f^{-1}} \Rightarrow f(4) + (-10) = 2 \Rightarrow K = -10$$

$$D_{f^{-1}} = R_f \quad \left\{ \begin{array}{l} f(v) = 3(v) - 10 = 11 \end{array} \right.$$

$$f(f(x)) = 3(3x - 10) - 10 = 9x - 40$$

$$(a, 2a) \in f \Rightarrow \frac{a(a)}{a+1} = 2a \Rightarrow a^2 = 2a^2 - 2a \Rightarrow a^2 - 2a = 0$$

از آنجا که اگر  $a=0$  آنگاه تابع یک به یک نخواهد بود داریم:

$$a(a-2) = 0 \Rightarrow a = 0 \times 2 \checkmark$$

$$\underline{a=2} \checkmark$$

الف)  $\left. \begin{array}{l} 5 \rightarrow 3 \rightarrow 4 \rightarrow 5 \checkmark \\ 7 \rightarrow 4 \rightarrow 3 \rightarrow 7 \checkmark \\ 2 \rightarrow 7 \rightarrow 5 \rightarrow 2 \checkmark \\ 6 \rightarrow 9 \rightarrow 8 \rightarrow 6 \checkmark \end{array} \right\} \{(5,5), (7,7), (2,2), (6,6)\}$

ب)  $\left. \begin{array}{l} 3 \rightarrow 5 \rightarrow 4 \rightarrow 3 \checkmark \\ 4 \rightarrow 7 \rightarrow 3 \rightarrow 4 \checkmark \\ 7 \rightarrow 2 \rightarrow 5 \rightarrow 7 \checkmark \\ 9 \rightarrow 6 \rightarrow 8 \rightarrow 9 \checkmark \end{array} \right\} \{(3,3), (4,4), (7,7), (9,9)\}$

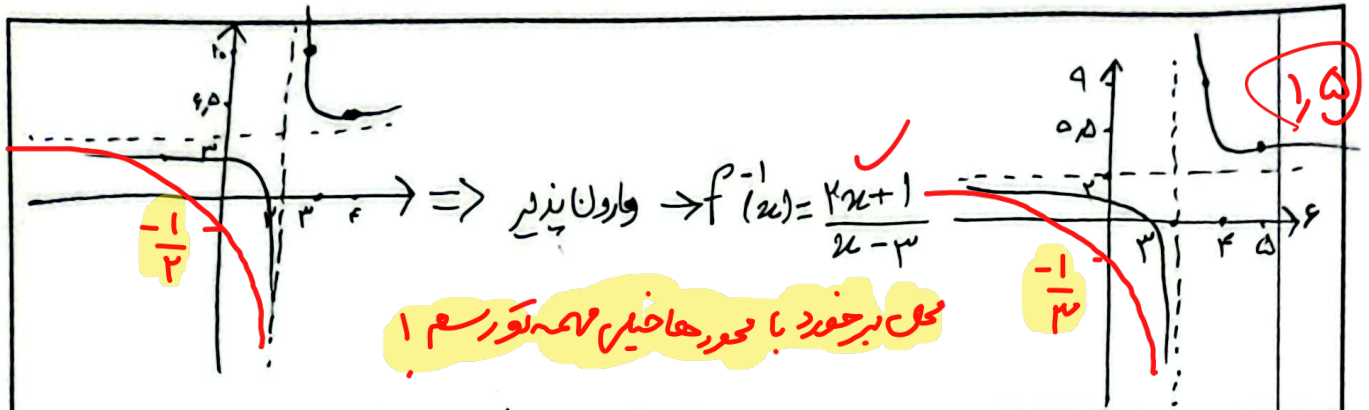
ج)  $\left. \begin{array}{l} 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \checkmark \\ 6 \rightarrow 8 \times \\ 5 \rightarrow 9 \rightarrow 7 \checkmark \end{array} \right\} \{(2,5), (5,4)\}$

د)  $\left. \begin{array}{l} 3 \rightarrow 5 \rightarrow 9 \checkmark \\ 4 \rightarrow 7 \times \\ 7 \rightarrow 2 \rightarrow 3 \checkmark \\ 9 \rightarrow 6 \rightarrow 8 \checkmark \end{array} \right\} \{(3,9), (7,3), (9,8)\}$

$D_h = \{1, 3, 4, 5\}$   $D_{g^{-1}} = \{1, 3, 5\}$   $\xrightarrow{① \text{ و } ②} \{1, 3, 5\} = \text{دامنه عبارت}$

$$\left. \begin{array}{l} x=1 \Rightarrow \frac{1 \rightarrow 2}{1 \rightarrow 2 \rightarrow 4} \Rightarrow \frac{2}{4} = \frac{1}{2} \checkmark \\ x=3 \Rightarrow \frac{3 \rightarrow 4}{3 \rightarrow 4 \rightarrow 1} \Rightarrow \frac{4}{1} = \frac{1}{4} \checkmark \end{array} \right\} \frac{h}{f \circ g^{-1}} = \frac{1}{2}$$

$x=5 \Rightarrow \frac{5 \rightarrow 1}{5 \rightarrow 1 \rightarrow 3} \Rightarrow \frac{1}{3} \times \text{نه}$



$$f(x) = |x-1| - |x-3| \quad -2 \leq x-4 \leq 2 \quad [a, b] \Rightarrow [1, 3]$$

$$f^{-1}(x) \Rightarrow x = 2y - 4 \Rightarrow 2y = x + 4 \Rightarrow y = \frac{x}{2} + 2; x \in [-2, 2]$$

$$x=1 \Rightarrow 1^3 + 4 = 5$$

$$x=0 \Rightarrow 4(0) - 1 = -1$$

هر دو تابع سکوی هستند بنابراین:

هیچگاه متداخل نداشته و وارون پذیرند.

$$f^{-1}(x) = \begin{cases} x = y^3 + 4 \Rightarrow y^3 = x - 4 \Rightarrow y = \sqrt[3]{x-4}; x \geq 4 \\ x = 4y - 1 \Rightarrow 4y = x + 1 \Rightarrow y = \frac{x+1}{4}; x \leq -1 \end{cases}$$

$$x^2 - \frac{x^3 + 3x^2 + 13x + 1}{x+3} = \frac{-3x+1}{x+3} \quad f^{-1}(x) = \frac{ax+b}{cx+d} \Rightarrow f(x) = \frac{-dx+b}{cx-a}$$

$$\frac{-4x+1}{2x+4} = \frac{-dx+b}{cx-a} \Rightarrow a = -4, d = +4, b = 1$$

$$f^{-1}(b) = \frac{-3b+1}{2b-4} \xrightarrow{b=1} \frac{-3+1}{2-4} = \frac{-2}{-2} = 1 \quad f^{-1}(b) = 1$$

$$y = \frac{x}{x^2+1} \Rightarrow yx^2 - x + y = 0 \Rightarrow x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$$

$$Df = R, Rf \Rightarrow x=1 \rightarrow \frac{1}{1+1} = \frac{1}{2} \max$$

$$x=-1 \rightarrow \frac{-1}{1+1} = -\frac{1}{2} \min \quad Rf = \left[-\frac{1}{2}, \frac{1}{2}\right]$$

$$f^{-1}(x) = \begin{cases} \frac{1-\sqrt{1-4x^2}}{2x}; x \in (0, \frac{1}{2}] \\ \frac{1+\sqrt{1-4x^2}}{2x}; x \in [-\frac{1}{2}, 0) \end{cases}$$