

$$f(x) \begin{cases} x^3 - x & ; x > a \rightarrow x^3 - x > a^3 - a \\ 12x - 20 & ; x \leq a \rightarrow 12a - 20 \leq 12a - 20 \end{cases}$$

$$\frac{a^3 - a}{a^3 - 12a + 14} \leq \frac{a^3 - x}{a^3 - 12a + 14} \rightarrow 0 \leq a^3 + 14 - 12a \rightarrow 0 \leq (a-2)^2 (a+1)$$

$$a \in [-1, +\infty)$$

$$\left. \begin{aligned} f(x) &= 2x + k \\ f^{-1}(x) &= x \rightarrow f(x) = 2 \end{aligned} \right\} \Rightarrow f(x) = 12 + k \Rightarrow k = -10 \Rightarrow f(x) = 2x - 10$$

الف) $f(x) = 2x - 10 \Rightarrow 11$ ✓

ب) $f(f(x)) = 9x - 10$ ✓

$$f(x) = \frac{ax}{x-1} \rightarrow f(a) = 2a \rightarrow \frac{a^2}{a-1} = 2a \rightarrow a^2 - 2a = 0$$

نسبت منفرجه غلط

$$\rightarrow a(a-2) = 0 \rightarrow a = 2$$

$$\Rightarrow f^{-1}(a) = a \rightarrow f(a) = 2a$$

$$f = \{(3, 5), (4, 7), (7, 2), (9, 6)\} \quad g = \{(3, 2), (1, 6), (9, 5)\}$$

$$f^{-1} = \{(5, 3), (7, 4), (2, 7), (6, 9)\} \quad g^{-1} = \{(2, 3), (6, 1), (5, 9)\}$$

الف) $f \circ f^{-1} = \{(5, 5), (7, 7), (2, 2), (6, 6)\}$ ج) $f \circ g^{-1} = \{(4, 5), (5, 6)\}$

ب) $f^{-1} \circ f = \{(3, 3), (4, 4), (7, 7), (9, 9)\}$ د) $g^{-1} \circ g = \{(3, 3), (1, 1), (9, 9)\}$

$$f = \{(3, 4), (4, 8), (9, 0)\} \quad g = \{(2, 1), (4, 3), (6, 5)\}$$

$$g^{-1} = \{(1, 2), (3, 4), (5, 6)\} \quad h = \{(1, 2), (3, 4), (4, 1), (5, 1)\}$$

$$\frac{h}{f \circ g^{-1}} = \{(1, \frac{2}{4}), (3, \frac{4}{8}), (5, \frac{1}{0})\}$$

با افق!

$y = \frac{x+1}{x-2}$ $\rightarrow$ تبدیل $\rightarrow y = \frac{2x+1}{x-2}$
 $\Rightarrow f^{-1}(x) = y = \frac{2x+1}{x-2}$

دقت! $(0, -\frac{1}{3})$

محل برخورد با محورهای مختصات

$x \in [1, 3] \xrightarrow{\text{تبدیل}} f(x) = 2x - 4 \rightarrow f^{-1}(x) = \frac{x}{2} + 2$
 $x = 2y - 4 \Rightarrow y = \frac{x}{2} + 2$

$-2 \leq x \leq 2$

همیشه وقت معلوم تابعی را بدست میدهی باید دامنه اش را مشخص کنی!

$f(x) = \begin{cases} x^2 + 4 & x \geq 1 \\ 4x - 1 & x \leq 0 \end{cases}$

تبدیل $\rightarrow$

$y = x^2 + 4; x \geq 1 \rightarrow x = \sqrt{y-4}; y \geq 5$
 $y = 4x - 1; x \leq 0 \rightarrow y = \frac{x+1}{4}; x \leq -1$

$f^{-1}(x) = \begin{cases} \sqrt{x-4} & x \geq 5 \\ \frac{x+1}{4} & x \leq -1 \end{cases}$

$f(x) = x^2 - \frac{(x+1)^2}{x+3} = \frac{x^3 + 3x^2 - x^2 - 2x - 1 - 2x}{x+3} = \frac{x^3 + 2x^2 - 4x - 1}{x+3}$

$\frac{-5x-1}{x+3} \xrightarrow{\frac{ax+b}{cx+d} \rightarrow \frac{a+bx}{a+dx}} \frac{-5x-1}{x+3} = f^{-1}(x) \Rightarrow f^{-1}(x) = \frac{-5x-2}{2x+9} \Rightarrow \begin{matrix} a=-5 \\ b=-2 \\ c=2 \end{matrix}$

$f^{-1}(b) = f^{-1}(-2) = \frac{-4x-2-2}{-2+9} = \frac{1}{7} = 5$

$f(x) = \frac{x}{x^2+1}$ وارن $\rightarrow x = \frac{y}{y^2+1} \rightarrow y^2x + x - y^2 = 0 \Rightarrow y^2 - y + x = 0$
 $\rightarrow y = \frac{1 \pm \sqrt{1-4x^2}}{2x} \xrightarrow{\text{ملاحظات}} \frac{1 + \sqrt{1-4x^2}}{2x} = y = f^{-1}(x)$

$\rightarrow R_f = [-0.5, 0.5]$ $D_f^{-1} = [-0.5, 0.5]$ $\rightarrow D_f = R \rightarrow R_f = R$

دقت! $\{0\}$

تبدیل $\rightarrow$ $D_f^{-1} = [-0.5, 0.5]$

نشان!