

$$f(x) = \begin{cases} x^3 - x & ; x > a \rightarrow x^3 - x > a^3 - a \\ 12x - 20 & ; x \leq a \rightarrow 12a - 20 \leq 12a - 20 \end{cases}$$

$\frac{a^3 - a}{a^3 - 12a + 14} \leq \frac{12a - 20}{a^3 - 12a + 14} \rightarrow 0 \leq a^3 + 14 - 12a \rightarrow 0 \leq (a-2)^2(a+1)$
 $\frac{a^3 - 12a + 14}{a^3 - 12a + 14} \leq \frac{12a - 20}{a^3 - 12a + 14} \rightarrow \frac{a^3 - 12a + 14}{a^3 - 12a + 14} \leq \frac{12a - 20}{a^3 - 12a + 14}$
 $a \in [-1, +\infty)$

$$\left. \begin{aligned} f(x) &= 2x + k \\ f^{-1}(x) &= x \rightarrow f(x) = 2 \end{aligned} \right\} \Rightarrow f(x) = 12 + k \Rightarrow k = -10 \Rightarrow f(x) = 2x - 10$$

الف) $f(x) = 2x - 10 \Rightarrow 11$

ب) $f(f(x)) = 9x - 10$

$$f(x) = \frac{ax}{x-1} \rightarrow f(a) = 2a \rightarrow \frac{a^2}{a-1} = 2a \rightarrow a^2 - 2a = 0$$

$\rightarrow a(a-2) = 0 \rightarrow a = 2$
 $\Rightarrow f^{-1}(a) = a \rightarrow f(a) = 2a$

$$f = \{(3, 5), (4, 7), (7, 2), (9, 6)\} \quad g = \{(3, 2), (1, 6), (9, 5)\}$$

$$f^{-1} = \{(5, 3), (7, 4), (2, 7), (6, 9)\} \quad g^{-1} = \{(2, 3), (6, 1), (5, 9)\}$$

الف) $f \circ f^{-1} = \{(5, 5), (7, 7), (2, 2), (6, 6)\}$ ج) $f \circ g^{-1} = \{(4, 5), (5, 6)\}$
 ب) $f^{-1} \circ f = \{(3, 3), (4, 4), (7, 7), (9, 9)\}$ د) $g^{-1} \circ g = \{(3, 3), (1, 1), (9, 9)\}$

$$f = \{(3, 4), (4, 8), (6, 0)\} \quad g = \{(2, 1), (4, 3), (6, 5)\}$$

$$g^{-1} = \{(1, 2), (3, 4), (5, 6)\} \quad h = \{(1, 2), (3, 4), (4, 1), (5, 1)\}$$

$\frac{h}{f \circ g^{-1}} = \{(1, \frac{2}{4}), (3, \frac{4}{8}), (5, \frac{6}{0})\}$

	$x \rightarrow y = \frac{x+1}{x-2} \rightarrow y+1 = \frac{x+1}{x-2} + 1 = \frac{x+1+x-2}{x-2} = \frac{2x-1}{x-2}$ $\Rightarrow f^{-1}(x) = y = \frac{x+1}{x-2}$	
	$x \in [1, \infty) \xrightarrow{\text{بديهي}} f(x) = x+1 \rightarrow f^{-1}(x) = \frac{x}{2} + 1$ $x = y+1 \Rightarrow y = \frac{x}{2} + 1 \leftarrow$	7
$f(x) = \begin{cases} x^2 + 1 & ; x \geq 1 \\ x-1 & ; x \leq 0 \end{cases}$		8
$f(x) = \begin{cases} x^2 - \frac{(x+1)^2}{x+2} & ; x \geq 1 \\ -\frac{5x-1}{x+2} & ; x \leq 0 \end{cases}$	$f(x) = \frac{x^2 - (x+1)^2}{x+2} = \frac{x^2 - x^2 - 2x - 1}{x+2} = \frac{-2x-1}{x+2}$ $f^{-1}(x) = \frac{-2x-1}{x+2}$	9
$f(x) = \frac{x}{x^2+1}$	$x = \frac{y}{y^2+1} \rightarrow y^2x + x - y^2 = 0 \Rightarrow y^2 - y + x = 0$ $\rightarrow y = \frac{1 \pm \sqrt{1-4x}}{2}$ $\rightarrow R_f = [-1/4, 1/4]$ $\rightarrow D_f = R \rightarrow R_f^{-1} = R$	10