

۲) به ازای هر $a \in \mathbb{R}$ درست است

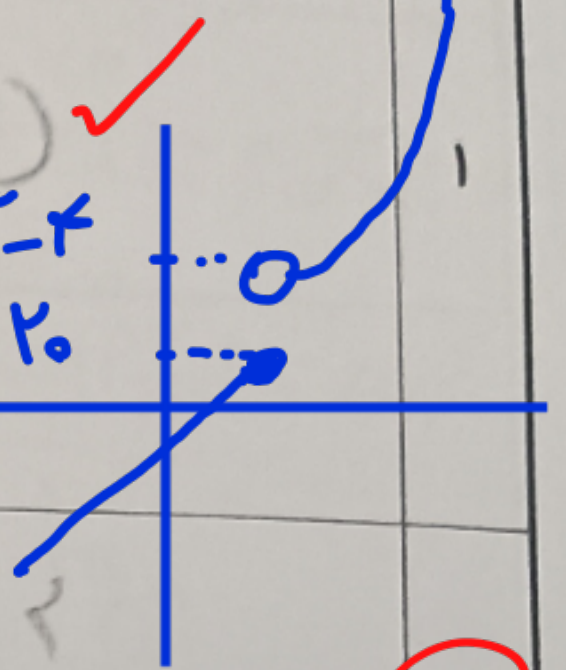
$$f(x) = \begin{cases} x^2 - 4, & x > a \\ x^2 - 4, & x < a \end{cases}$$

$a \in [-4, +\infty)$ ✓

$a^2 - 4$

$\rightarrow x^2 - 4 \geq 1 \Rightarrow x^2 \geq 5 \Rightarrow x \geq \sqrt{5} \text{ or } x \leq -\sqrt{5}$

$\frac{-4}{-1+1+}$



$f^{-1}(1) = \{x \mid x^2 - 4 = 1\} \Rightarrow x^2 = 5 \Rightarrow x = \pm\sqrt{5}$

$f(1) = 1^2 - 4 = -3 \Rightarrow x^2 - 4 = -3 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

$\rightarrow x^2 = 1 \Rightarrow x = \pm 1$ ✓

$f(x) = x^2 - 4$ ✓

$f(f(x)) = f(x^2 - 4) = (x^2 - 4)^2 - 4 = x^4 - 8x^2 + 12$ ✓

$y = \frac{ax}{x-1} \xrightarrow{\text{دارد}} x = \frac{ay}{y-1} \rightarrow yx - x = ay \rightarrow yx - ay = x \rightarrow y(x-a) = x$

$\rightarrow y = \frac{x}{x-a} \rightarrow \frac{1}{y} = \frac{x-a}{x} = 1 - \frac{a}{x} \Rightarrow \frac{1}{y} - 1 = -\frac{a}{x} \Rightarrow \frac{1-y}{y} = -\frac{a}{x} \Rightarrow x = \frac{ay}{1-y}$ ✓

۲) $f \circ f^{-1} = \{(1, 1), (4, 4), (9, 9), (16, 16)\}$ ✓

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۳) $f^{-1} \circ f = \{(1, 1), (4, 4), (9, 9), (16, 16)\}$ ✓

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۴) $f \circ g^{-1} = \{(1, 1), (4, 4)\}$ ✓

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۵) $g^{-1} \circ f = \{(1, 1), (4, 4), (9, 9)\}$ ✓

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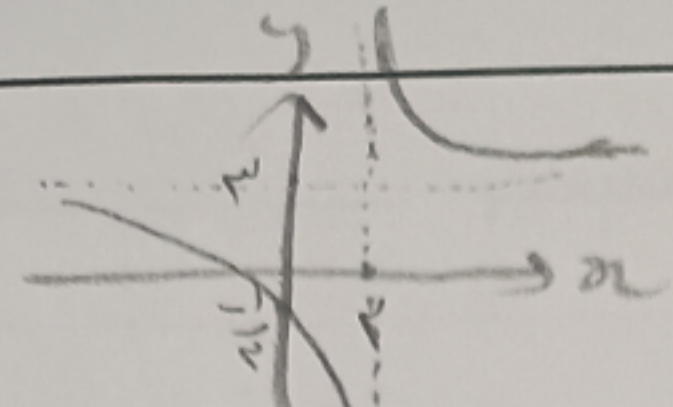
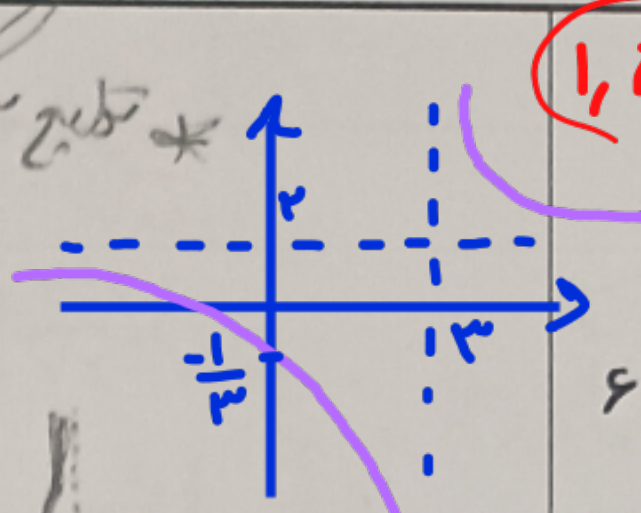
$g^{-1} = \{(1, 1), (4, 4), (9, 9)\}$

$f \circ g^{-1} = \{(1, 1), (4, 4), (9, 9)\}$

$D_h \cap D_{f \circ g^{-1}} = \{1, 4, 9\}$

$\frac{1}{f \circ g^{-1}} = \{(1, \frac{1}{1}), (4, \frac{1}{4}), (9, \frac{1}{9})\}$

$\Rightarrow \frac{1}{f \circ g^{-1}} = \{(1, \frac{1}{1}), (4, \frac{1}{4})\}$ ✓

$y = \frac{x+1}{x-2} \rightarrow$  $\rightarrow$ 

$6x = \frac{x^2+1}{x-2} \rightarrow yx + 2x = x^2 + 1 \rightarrow yx - x^2 = 1 - 2x$

$\rightarrow y(x-2) = 1-2x \rightarrow y = \frac{1-2x}{x-2}$ (فقط!)

* تابع معکوس ندارد ✓

$f(x) = |x-1| - |x-2|$

$\begin{cases} x > 2 \rightarrow x-1-x+2 = 1 & x > 2 \rightarrow \text{معکوس ندارد} \\ 1 < x < 2 \rightarrow x-1-x+2 = 1 & 1 < x < 2 \rightarrow \text{معکوس دارد} \\ x < 1 \rightarrow x+1-x-2 = -1 & x < 1 \rightarrow \text{معکوس ندارد} \end{cases}$

$\boxed{a} \quad \boxed{b}$

$\rightarrow x-2 \leq y \rightarrow x-2 = y \rightarrow x = y+2 \rightarrow y = \frac{x+2}{2} = \frac{x}{2} + 1$

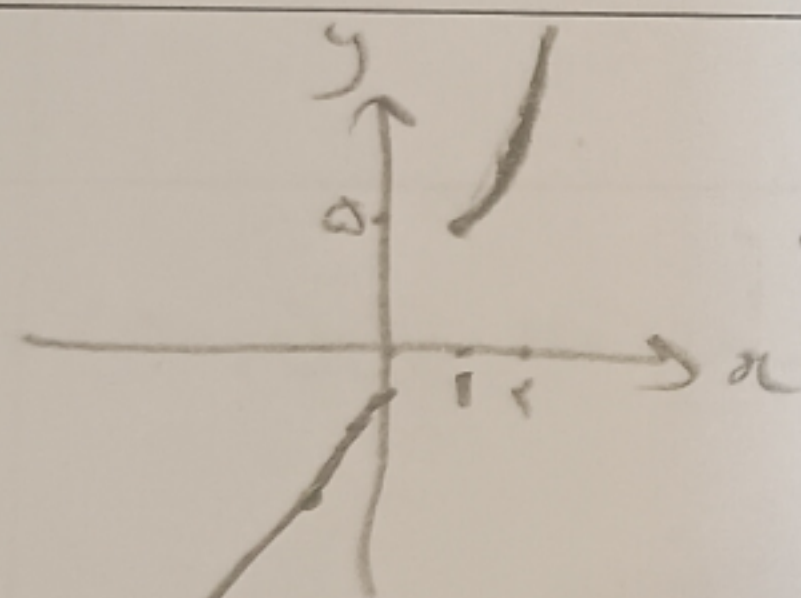
همیشه وقتی معلوم تابعی را بدست می‌دهی باید دامنه اش را مشخص کنی!

$-2 \leq x \leq 2$

$y = \sqrt{x-1} \rightarrow y^2 = x-1 \rightarrow x = y^2+1$

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$y = \frac{x^2 - (x+1)^2}{x+2} = \frac{x^2 - (x^2+1+2x(x+1))}{x+2} = \frac{x^2 + x^2 - x^3 - 1 - 2x(x+1)}{x+2}$

$\frac{x^2-1}{x+2} \rightarrow x = \frac{-2y-1}{y+2} \rightarrow yx + 2x = -2y-1 \rightarrow yx + 2y = -1-2x$

$\rightarrow y(x+2) = -1-2x \rightarrow y = \frac{-1-2x}{x+2}$

$f^{-1}(-5) = \frac{12-2}{-5+2} = \frac{10}{-3} = -\frac{10}{3}$

$f(x) = \frac{x}{x^2+1} \rightarrow y = \frac{x}{x^2+1} \rightarrow yx^2 + y = x \rightarrow yx^2 - x + y = 0 \rightarrow yx^2 - x + y = 0$

$\rightarrow 0 = 1-4y^2 \rightarrow x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$

$f^{-1}(y) = \frac{1 \pm \sqrt{1-4y^2}}{2y}$

$|x| \leq \frac{1}{y} - \{0\}$