

بازای $x \in \mathbb{R}$ درست است

$$f(x) = \begin{cases} x-4, & x > 4 \\ x-10, & x \leq 4 \end{cases}$$

$a \in [-4, +\infty)$

$$\rightarrow x-4 = x-10 \rightarrow x-4 = x-10 \rightarrow \frac{x-4}{x-4} = 1$$

$f^{-1}(x) = \begin{cases} x-4, & x > 4 \\ x-10, & x \leq 4 \end{cases} \rightarrow f(x) = \begin{cases} x-4, & x > 4 \\ x-10, & x \leq 4 \end{cases}$

$\rightarrow x-4 = x-10$

$$f(f(x)) = f(x-10) = (x-10)-10 = x-20$$

$y = \frac{ax}{x-1} \rightarrow x = \frac{ay}{y-1} \rightarrow yx - x = ay \rightarrow yx - ay = x \rightarrow y(x-a) = x$

$\rightarrow y = \frac{x}{x-a} \rightarrow \frac{1}{y} = \frac{x-a}{x} = 1 - \frac{a}{x} \rightarrow \frac{1}{y} = 1 - \frac{a}{x} \rightarrow \frac{1}{y} = 1 - \frac{a}{x}$

الف) $f \circ f^{-1} = \{(1,1), (2,2), (3,3), (4,4)\}$

ب) $f^{-1} \circ f = \{(1,1), (2,2), (3,3), (4,4)\}$

ج) $f \circ g^{-1} = \{(1,1), (2,2)\}$

د) $g^{-1} \circ f = \{(1,1), (2,2), (3,3)\}$

$g^{-1} = \{(1,1), (2,2), (3,3)\}$

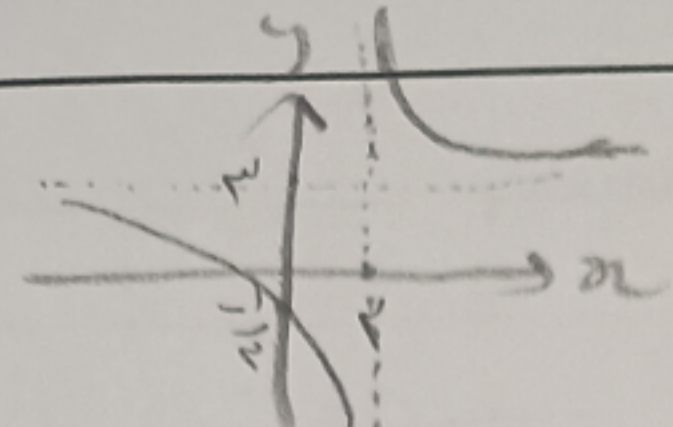
$f \circ g^{-1} = \{(1,1), (2,2), (3,3)\}$

$D_h \cap D_{f \circ g^{-1}} = \{1, 2, 3\}$

تساوی

$\frac{1}{f \circ g^{-1}} = \{(1, \frac{1}{1}), (2, \frac{1}{2}), (3, \frac{1}{3})\}$

$\Rightarrow \frac{1}{f \circ g^{-1}} = \{(1, \frac{1}{1}), (2, \frac{1}{2}), (3, \frac{1}{3})\}$

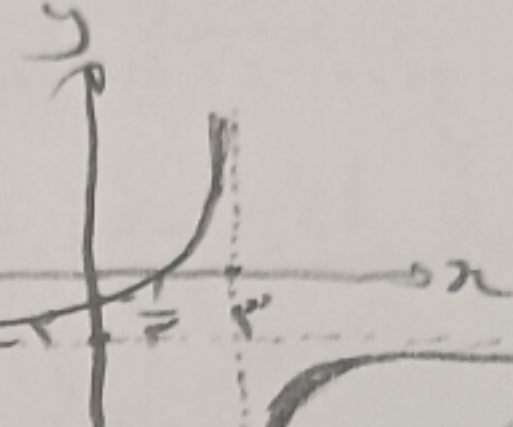
$y = \frac{x+1}{x-2} \rightarrow$ 

$6x = \frac{x^2+1}{x-2} \rightarrow yx + 2x = x^2 + 1 \rightarrow yx - x^2 = 1 - 2x$

$\rightarrow y(x-2) = 1-2x \rightarrow y = \frac{-2x+1}{x-2}$

* تابع معکوس پیدا می‌کنیم

$y = \frac{x+1}{x-2} \rightarrow y(x-2) = x+1 \rightarrow yx - 2y = x+1 \rightarrow yx - x = 2y+1 \rightarrow x(y-1) = 2y+1 \rightarrow x = \frac{2y+1}{y-1}$

$\rightarrow y = \frac{x+1}{x-2}$ 

$f(x) = |x-1| - |x-2|$

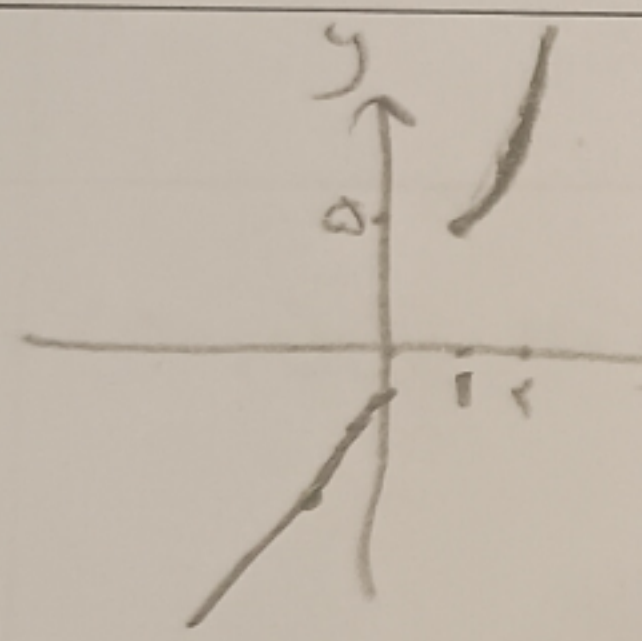
$\begin{cases} x > 2 \rightarrow x-1 - (x-2) = 1 \\ 1 < x < 2 \rightarrow x-1 - (2-x) = x-1 \\ x < 1 \rightarrow -(x-1) - (2-x) = -1 \end{cases}$

$x > 2 \rightarrow$ معکوس ندارد

$1 < x < 2 \rightarrow$ معکوس دارد ✓

$x < 1 \rightarrow$ معکوس ندارد

$\boxed{1 < x < 2} \rightarrow$ $x-1 = y \rightarrow x = y+1 \rightarrow y+1-2 = -y \rightarrow y = \frac{y+1-2}{-1} = \frac{y-1}{-1} = 1-y$



$y = \sqrt{x-1} \rightarrow y^2 = x-1 \rightarrow x = y^2+1$

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$y = \frac{x^2 - (x+1)^2}{x+2} = \frac{x^2 - (x^2 + 2x + 1)}{x+2} = \frac{-2x-1}{x+2}$

$\frac{-2x-1}{x+2} = y \rightarrow -2x-1 = y(x+2) \rightarrow -2x-1 = xy+2y \rightarrow -2x-xy = 2y+1 \rightarrow x(-2-y) = 2y+1 \rightarrow x = \frac{2y+1}{-2-y}$

$\rightarrow y = \frac{x^2 - (x+1)^2}{x+2}$

$f^{-1}(y) = \frac{12-2}{-2-y} = \frac{10}{-2-y}$

$f(x) = \frac{x}{x^2+1} \rightarrow y = \frac{x}{x^2+1} \rightarrow yx^2 + y = x \rightarrow yx^2 - x + y = 0 \rightarrow yx^2 - x + y = 0$

$\rightarrow x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$

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