

$$12a - 20a^3 - 4$$

$$a^3 - 12a + 14 \geq 0$$

$$a^3 - 12a + 14 = (a + 4)(a - 2)^2$$

$$\begin{array}{c|c|c|c} -4 & 2 & & \\ \hline - & + & + & \\ & 2 & 2 & 2 \end{array}$$

$$\Rightarrow a \geq -4$$

* نباید یکدیگر را جمع کرد
 $a^3 - 4, 12a - 20$

$$f^{-1}(x) = 2 \Rightarrow f(2) = 2$$

$$f(x) = 3x + k \Rightarrow f(2) = 3(2) + k = 2$$

$$k = -4$$

$$f(v) =$$

$$3(v) - 10 = 11$$

$$f(f(x)) =$$

$$f(3x - 10) =$$

$$3(3x - 10) - 10 =$$

$$9x - 30 - 10 =$$

$$9x - 40$$

$$f(x) = \frac{ax}{x-1} \Rightarrow f^{-1}(x) = y = \frac{ay}{y-1} = xy - x = ay \Rightarrow xy - ay = x$$

$$y(x-a) = x \Rightarrow y = \frac{x}{x-a} \xrightarrow{(x,a)} a = \frac{xa}{xa-a} \Rightarrow a = \frac{xa}{a} = x$$

$$\text{الف) } f(f^{-1}(x)) = \{(1,1), (2,2), (3,3), (4,4)\}$$

$$\text{ب) } f^{-1}(f(x)) = \{(1,1), (2,2), (3,3), (4,4)\}$$

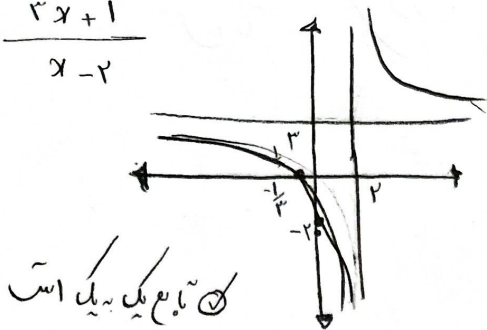
$$\text{ج) } f(g^{-1}(x)) = \{(1,2), (2,3)\}$$

$$\text{د) } g^{-1}(f(x)) = \{(1,3), (2,4), (3,5)\}$$

$$f(g^{-1}(x)) = \{(1,2), (2,3), (3,4)\} \quad \frac{h}{f \circ g^{-1}}, \left\{ \frac{2}{2}, \frac{3}{3} \right\} = \left\{ \frac{1}{1} \right\}$$

$$\left(\frac{h}{f \circ g^{-1}} \right)^A \rightarrow D_A = D_h \cap D_{f \circ g^{-1}} \mid D_{f \circ g^{-1}} \neq \emptyset \Rightarrow \{1, 2, 3, 4\} \cap \{1, 2\} = \{1, 2\}$$

$$y = \frac{3x+1}{x-2}$$

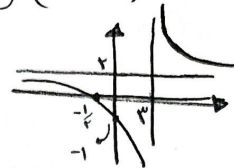


سویچ کردیم

$$\Leftrightarrow x = \frac{3y+1}{y-2} = 3y - 2x = 3y+1 \Rightarrow$$

$$xy - 3y = 1 + 2x = y(x-3) = 1+2x$$

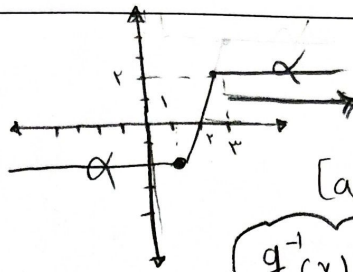
$$y = \frac{3x+1}{x-3}$$



$$f(x) = |x-1| - |x-3|$$

$$\frac{-x+1+x-3}{-2} \quad \frac{x-1+x-3}{2x-4} \quad \frac{x+1-x+3}{4}$$

$$f(x) = \begin{cases} x \geq 2 & 2 \\ 1 < x < 2 & 2x-4 \\ x < 1 & -2 \end{cases}$$



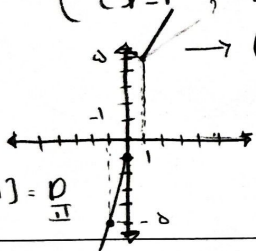
$$[a, b] = [1, 2]$$

$$g^{-1}(x) = y = \frac{x+2}{2}; x \in [-2, 2]$$

$$g(x) = 2x-4 \rightarrow g^{-1}(x) = x = 2y-4 \Rightarrow 2y = x+4$$

$$f(x) = \begin{cases} x^2 + 2; x \geq 1 & \text{I} \\ 2x-1; x < 1 & \text{II} \end{cases} \xrightarrow{\text{I, II}} f^{-1}(x)$$

$$\begin{cases} \sqrt{x-2}; x \in [2, +\infty) \\ \frac{x+1}{2}; x \in (-\infty, 1] \end{cases}$$



$$R_{II} = (-\infty, -1] = D_{II}$$

سویچ کردیم

$$f(x) = x^2 - \frac{(x+1)^2}{x+2}$$

$$\begin{cases} 0 \rightarrow -\frac{1}{2} \\ 0 \leftarrow -\frac{1}{2} \end{cases}$$

$$\begin{cases} 1 \rightarrow -1 \\ 1 \leftarrow -1 \end{cases}$$

$$f^{-1}(x) = \frac{ax+b}{2x+d}$$

$$\frac{-a}{2} + b = 0 \Rightarrow a = 2b$$

$$\begin{aligned} -a+b &= -2+d \\ -2b-d &= -2 \end{aligned}$$

$$f^{-1}(-2) = \frac{1-2}{2-2+d} = 1$$

$$f^{-1}(-2) = \frac{-2a-2}{2a+2} = 1 \Rightarrow \begin{cases} b=-2 \\ a=-2 \end{cases}$$

$$f(x) = \frac{x}{x^2+1} \xrightarrow{\text{I, II}} x = \frac{y}{y^2+1} = y^2y + y = y \Rightarrow xy^2 - y + x = 0$$

$$y = \frac{1 \pm \sqrt{1-4x^2}}{2x}$$

$$y = \frac{1 \pm \sqrt{1-4x^2}}{2x}; x \in [-\frac{1}{2}, 0) \cup (0, \frac{1}{2}]$$

$$\sqrt{1-4x^2} \neq 0 \Rightarrow 1-4x^2 \neq 0 \Rightarrow x^2 \leq \frac{1}{4} \Rightarrow -\frac{1}{2} \leq x \leq \frac{1}{2}$$