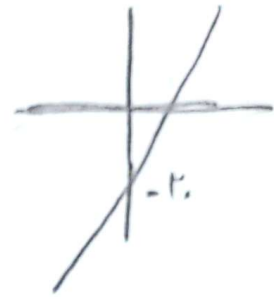
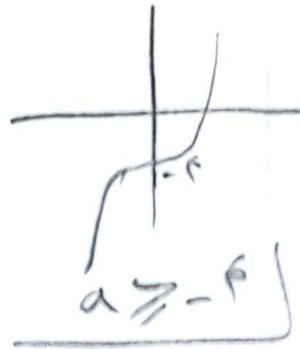


مرد و زن به یکدیگر یک به یک هستند

$$x^3 - 4 \geq 12x - 20$$

$$x^3 - 12x + 16 \geq 0$$

$$(x-2)(x-2)(x+4) \geq 0$$



۱

$$f^{-1}(2) = 4 \rightarrow f(4) = 2 \rightarrow f(4) = 3x + k \rightarrow 2 = 12 + k \rightarrow k = -10$$

$$f(x) = 3x - 10$$

$$الف) f(7) = 3 \times 7 - 10 = 11$$

۲

$$ب) f(f(x)) = f(3x - 10) = 3(3x - 10) - 10 = 9x - 40$$

$$f^{-1}(2a) = a \rightarrow f(a) = 2a \rightarrow f(x) = \frac{ax}{x-1}$$

$$\rightarrow 2a = \frac{a^2}{a-1} \rightarrow 2a^2 - 2a = a^2 \rightarrow a^2 - 2a = 0$$

$$\rightarrow a(a-2) = 0 \rightarrow a = 0 \text{ (مرد و زن به یکدیگر یک به یک هستند)} \rightarrow a = 2 \checkmark$$

۳

$$الف) \{(4,4), (7,7), (2,2), (6,6)\}$$

$$ب) \{(3,3), (4,4), (7,7), (9,9)\}$$

$$ج) \{(4,4), (2,2)\}$$

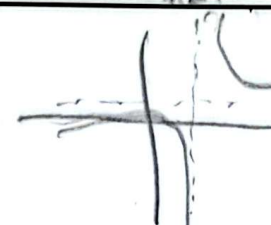
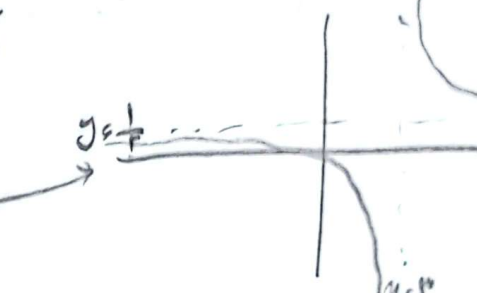
$$د) \{(7,7), (3,3), (9,9)\}$$

۴

$$\frac{h}{f \circ g^{-1}} = \left\{ \left(1, \frac{2}{3}\right), \left(3, \frac{4}{1}\right), \left(5, \frac{1}{1}\right) \right\}, \left\{ \left(1, \frac{1}{4}\right), \left(3, \frac{1}{4}\right) \right\}$$

$$ج) \{(1,2), (3,4), (5,6)\}$$

۵

یک به یک است $g \circ f$  $x = \frac{y-1}{y-2} = \frac{y+1}{y-2} \rightarrow y = \frac{x+1}{x-2}$ 	۶
 $a=1, b=2 \rightarrow y=2x-1 \rightarrow y+1=2x$ $\rightarrow \frac{y}{2}+1=x \rightarrow y=\frac{x}{2}+2$	۷
 $y=x^2+1 \rightarrow y-1=x^2 \rightarrow x=\sqrt{y-1} \rightarrow y=\sqrt{x-1}+1$ $y=x^2-1 \rightarrow y+1=x^2 \rightarrow x=\frac{y+1}{2} \rightarrow y=\frac{x+1}{2}$ $f(x)=\begin{cases} \sqrt{x-1}+1 & ; x \geq 1 \\ \frac{x+1}{2} & ; x \leq 0 \end{cases}$	۸
$x^2 - \frac{(x+1)^2}{x+2} = \frac{x^2(x+2) - (x+1)^2}{x+2} = \frac{-x^2-1}{x+2} = \frac{-x-1}{x+2}$ $a=-4, b=-2, d=3$ $f'(b) = f'(-2) = \frac{-2(-2)-1}{(-2)+3} = \frac{3}{1} = 3$	۹
$y = \frac{x}{x^2+1} \rightarrow yx^2+x+y=0 \rightarrow x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$ $\Rightarrow y = \frac{1 \pm \sqrt{1-4x^2}}{2x}$	۱۰