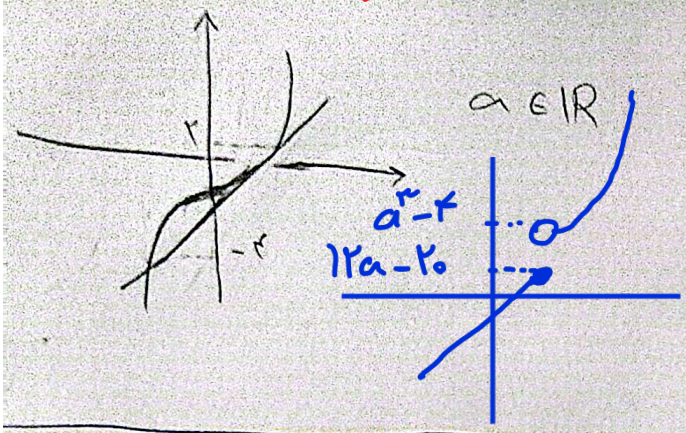
۱



$a^3-4 \geq 12a-20$

$(a-2)^2(a+4) \geq 0$

$-4 \quad 2$

$- \mid + \mid +$

$a \geq -4$ ✓

۵

۲ $x = 2y+k \Rightarrow 2y = x-k \rightarrow y = \frac{1}{2}x - \frac{k}{2} = f^{-1}(x) \Rightarrow f^{-1}(2) = \frac{2}{2} - \frac{k}{2} = 6 \Rightarrow k = -10$ ✓

$\Rightarrow f(x) = 2x-10 \quad f(7) = 2(7)-10 = 14-10 = 11$ ✓

$f(f(x)) = f(2x-10) = 2(2x-10)-10 = 9x-20-10 = 9x-30$ ✓

۲

۳ $x = \frac{ay}{y-1} \xrightarrow{(2a,a)} 2a = \frac{a^2}{a-1} \Rightarrow 2a^2-2a = a^2 \Rightarrow a^2-2a = 0 \Rightarrow a = 0 \text{ غ ق ق}, 2$ ✓

۱٫۷۵

$a = 0 \rightarrow f_{(x)} = 0$ تابع ثابت صفر وارون پذیر نیست!

۴ الف) $fof^{-1} = \{(5,5)(7,7)(2,2)(6,6)\}$ ✓

$f^{-1}of = \{(2,2)(4,4)(7,7)(9,9)\}$ ✓

۱٫۵

ب) $fog^{-1} = \{(2,5)(a,6)\}$ ✓

$g^{-1}of$: ~~$\emptyset$~~

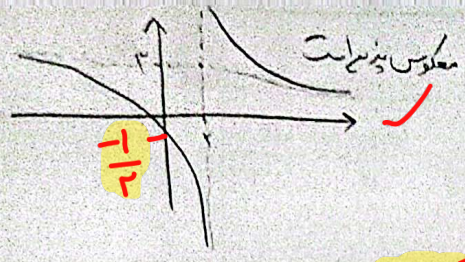
$\{(6,9)(7,6)(9,8)\}$

۵ $fog^{-1} = \{(1,6)(2,8)(a,0)\} \Rightarrow \frac{h}{fog^{-1}} = \{(1,1)(2,1)(a,\frac{1}{4})\}$ ~~(1,1)(2,1)(a,1/4)~~

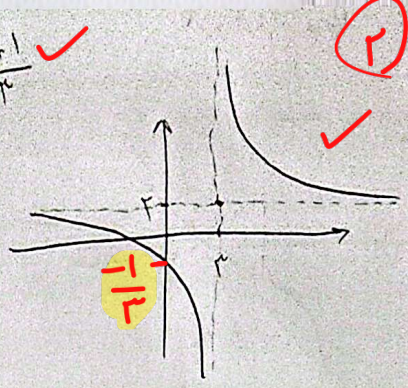
۱٫۲۵

$\{(\frac{1}{2},2)(1,\frac{1}{2})\}$

۶ (۲)

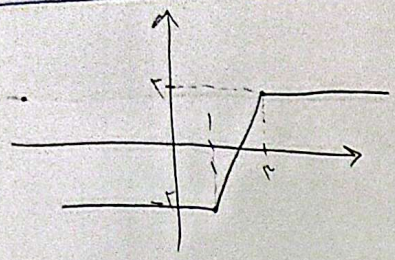


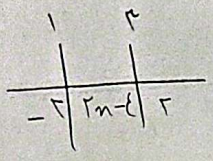
$x = \frac{2y+1}{y-2} \Rightarrow y = \frac{2x+1}{x-2}$ ✓



محل برخورد با محورها خیلی مهمه تو رسم!

۷ (۱,۷۵)



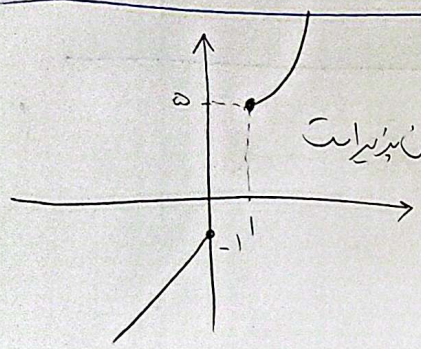


$\Rightarrow \begin{cases} a=1 \\ b=2 \end{cases}$ ✓

همیشه وقتی معکوس تابعی را بدست میدی باید دامنه اش را مشخص کنی!

$x = 2y - 4 \Rightarrow 2y = x+4 \Rightarrow y = \frac{1}{2}x + 2 = f^{-1}(x)$ ✓ $\quad D = [-2, 2]$

۸ (۲)



وارون پذیر است

$x = y^2 + 4 \rightarrow y^2 = x-4 \Rightarrow y = \sqrt{x-4}$

$\xrightarrow{y \geq 1} \sqrt{x-4} \geq 1 \Rightarrow x-4 \geq 1 \Rightarrow x \geq 5$

$x = 4y - 1 \rightarrow 4y = x+1 \Rightarrow y = \frac{x+1}{4}$

$\xrightarrow{y \leq 0} \frac{x+1}{4} \leq 0 \Rightarrow x+1 \leq 0 \Rightarrow x \leq -1$

$\Rightarrow f^{-1}(x) = \begin{cases} \sqrt{x-4} & x \geq 5 \\ \frac{x+1}{4} & x \leq -1 \end{cases}$ ✓

۹ (۲)

$f(x) = \frac{x^4 + 2x^3 - x^2 - 2x - 1}{x+3} = \frac{-2x-1}{x+2}$

$x = \frac{-2y-1}{y+2} \Rightarrow y = \frac{-2x-1}{x+2} = \frac{-2x-2}{2x+4} \Rightarrow \begin{cases} a=-2 \\ b=-2 \\ d=4 \end{cases}$

$\Rightarrow f^{-1}(-2) = \frac{-4(-2)-2}{2(-2)+4} = \frac{10}{2} = 5$ ✓

۱۰ (۱,۷۵)

$x = \frac{y}{y^2+1} \Rightarrow y^2 x + x = y \Rightarrow y^2 x - y + x = 0 \quad y = \frac{1 \pm \sqrt{1-4x^2}}{2x}$ ✓

$|x| \leq \frac{1}{2} - \{0\}$