نام و نام خانوادگی فرهاد فیروزبنده .... پاسخنامه تشریحی تکلیف شماره ۱۳ کلاس یازدهم تجربی B

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$y = (a-1)x^2 + x + 3$

رأس $x = \frac{-b}{2a} = 2 = \frac{-1}{2(a-1)} = 4a - 4 = -1 \Rightarrow a = \boxed{+\frac{3}{4}}$

$y = -\frac{1}{4}x^2 + x + 3 \xrightarrow{y=0} -\frac{1}{4}x^2 + x + 3 = 0 \Rightarrow x^2 - 4x - 12 = 0$

$\rightarrow x = \frac{4 \pm 8}{2}$ ، $x_1 = 6 \rightarrow \boxed{x = 6}$ ، $x_2 = -2$ غ ق ق

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$f(x) = mx^2 - 5x + m$

$\Delta = (-5)^2 - 4m^2 = 25 - 4m^2 \xrightarrow{\Delta > 0} 25 - 4m^2 > 0 \rightarrow 4m^2 < 25 \rightarrow |m| < \frac{5}{2}$

$\xrightarrow{m<0} -\frac{5}{2} < m < 0 \longrightarrow \boxed{-\frac{5}{2} < m < 0} \longrightarrow$

	-2.5		2.5	
$-$		$+$		$-$

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$x_1 = \frac{3+\sqrt{5}}{3}, \quad x_2 = \frac{3-\sqrt{5}}{3} \longrightarrow x^2 - (x_1 + x_2)x + x_1x_2 = 0$

$\longrightarrow x_1 + x_2 = 2, \quad x_1x_2 = \frac{4}{9} \longrightarrow x^2 - 2x + \frac{4}{9} = 0$

$\xrightarrow{\times 9} 9x^2 - 18x + 4 = 0$

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$2x^2 - 8x + m + 2 = 0$

$3\alpha^2 + \beta^2 = 12 \rightsquigarrow 3\alpha^2 + (4-\alpha)^2 = 12 \rightarrow 4\alpha^2 - 8\alpha + 4 = 0 \xrightarrow{\Delta} \alpha = 1$

$\alpha + \beta = \frac{-b}{a} = \frac{8}{2} = 4 \longrightarrow \beta = 4 - \alpha$

$\hookrightarrow \beta = 4 - \alpha \Rightarrow \beta = 3$

$\longrightarrow \alpha < \beta$

$\alpha\beta = \frac{c}{a} = \frac{m+2}{2} \rightsquigarrow \alpha\beta = 1 \times 3 = 3 = \frac{m+2}{2} \Rightarrow \boxed{m = 4}$

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رأس $\longrightarrow (2, 9) \longrightarrow (h, k)$

$y = a(x-h)^2 + k \longrightarrow y = a(x-2)^2 + 9 \rightsquigarrow -(x-2)^2 + 9$

$\longrightarrow x - 2 = \pm 3$ ، $x_1 = -1$ ، $x_2 = 5$

$\longrightarrow a(5-2)^2 + 9 = 0 \longrightarrow 9a + 9 = 0 \Rightarrow a = -1$

$\rightsquigarrow -1 < x < 5$ یا $(-1, 5)$

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$\alpha x^2 - 7x + 9\beta = 0$

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{1}{-3} + \frac{3}{2} = \boxed{\frac{7}{6}}$

$x_1 + x_2 = \frac{-b}{a} = \frac{7}{a}$

$x_1 x_2 = \frac{c}{a} \rightsquigarrow \frac{9\beta}{a} \to \alpha\beta = \frac{9\beta}{\alpha} = \alpha^2 = a \to a = \pm 3$ قبول $+$ $\to a = -3$, $\beta = \frac{2}{3}$

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$x^2 + mx - 2m = 0$ $\quad a + \beta = -m = \begin{cases} -2 \\ 4 \end{cases} \longleftarrow m = \frac{-2 \pm 6}{2} \begin{cases} m_1 = 2 \\ m_2 = -4 \end{cases}$

$a^2 - m\beta = 8$

$a + \beta = -m \to m = -(a+\beta) \xrightarrow{\text{جایگذاری}} a^2 - (-(a+\beta))\beta = 8.$

$\to a^2 + \beta(a+\beta) = 8 \to a^2 + a\beta + \beta^2 = 8$

$\alpha\beta = -2m$

$\to (a+\beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2 \to (a+\beta)^2 - a\beta = 8 \to m^2 + 2m - 8 = 0$

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$f(x) = mx^2 + 4x + \frac{m}{2} + 7$

رأس $x = \frac{-b}{2a} \to \frac{-4}{m} = \frac{-2}{m} = k \to F(k) = 8 \to F(-\frac{2}{m}) = m\left(\frac{4}{m^2}\right) + 4\left(-\frac{2}{m}\right) + \frac{m}{2} + 7$

$\to \frac{4}{m} - \frac{8}{m} + \frac{m}{2} + 7 \to F(-\frac{2}{m}) = -\frac{4}{m} + \frac{m}{2} + 7 = 8 \to -\frac{4}{m} + \frac{m}{2} = 1$

$\to -8 + m^2 = 2m \to m^2 - 2m - 8 = 0$

$m = \frac{2 \pm \sqrt{32+4}}{2} = \frac{2 \pm 6}{2} \begin{cases} 4 & \text{غ ق ق} \\ -2 & \text{ق ق} \end{cases} \to f(x) = -2x^2 + 4x + 6 \to x = -1, 3$

$\to \boxed{x = 3}$

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$f(x) = a(x-2)^2 + 6 \xrightarrow{(0,4)} 4 = a(0-2)^2 + 6 = 4 \to 4a + 6 \to a = -\frac{1}{2}$

$f(x) = -\frac{1}{2}(x-2)^2 + 6 \xrightarrow{f(x)=0} -\frac{1}{2}(x-2)^2 + 6 = 0 \to (x-2)^2 = 12 \begin{cases} x_1 = 2 + 2\sqrt{3} \\ x_2 = 2 - 2\sqrt{3} \end{cases}$

$\to \frac{1}{x_1} + \frac{1}{x_2} = \frac{x_1 + x_2}{x_1 x_2} = \frac{2 + 2\sqrt{3} + 2 - 2\sqrt{3}}{4 - 12} = \frac{4}{-8} = \boxed{-\frac{1}{2}}$

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$x^2 - (4a+4)x + (3a^2 + 6a + 3) = 0$

$\to x_1 + x_2 = 4a + 4 \quad \xrightarrow{x_1 = 2} \quad 2 + x_2 = 4a + 4 \to x_2 = 4a + 2$

$\to x_1 x_2 = 3a^2 + 6a + 3 \quad\quad 2(4a+2) = 3a^2 + 6a + 3 \Rightarrow$

$\to 8a + 4 = 3a^2 + 6a + 3 \to 3a^2 - 2a - 1 = 0 \to \frac{2 \pm \sqrt{4+12}}{6}$

$\begin{cases} a_1 = 1 \Rightarrow x_2 = 6 \\ a_2 = -\frac{1}{3} \Rightarrow x_2 = \frac{2}{3} \end{cases}$ $\rightsquigarrow$ ریشه دیگر می تواند $\to \boxed{6, \frac{2}{3}}$