۱

$\Rightarrow -\frac{b}{2a} = 2 \Rightarrow b = -4a \Rightarrow -4(a-1) = 1 \Rightarrow a = \frac{3}{4} \Rightarrow y = -\frac{1}{4}x^2 + x + 3$

$\Rightarrow x = \frac{-1 \pm \sqrt{1 - 4(\frac{1}{4})(3)}}{-\frac{1}{2}} = \frac{-1 \pm \sqrt{1+3}}{-\frac{1}{2}} = 2 \pm 2 = \begin{cases} 4 \\ \text{غ ق ق} \end{cases}$

۲

$m < 0$ ① $\quad \Delta > 0 \Rightarrow 25 - m^2 > 0 \Rightarrow 25 > m^2 \Rightarrow -5 < m < 5$ ②

① ∩ ② $\rightarrow m \in (-5, 0)$

۳

$S = \frac{3 - \sqrt{5} + 3 + \sqrt{5}}{3} = \frac{6}{3} = 2 \quad P = \frac{3 - \sqrt{5}}{6} \times \frac{3 + \sqrt{5}}{6} = \frac{9 - 5}{9} = \frac{4}{9}$

$x^2 - Sx + P = 0 \Rightarrow x^2 - 2x + \frac{4}{9} = 0 \Rightarrow 9x^2 - 18x + 4 = 0$

۴

$3\alpha^2 + \beta^2 = 2\alpha^2 + 2\beta^2 + \alpha^2 - \beta^2 = 2(\alpha^2 + \beta^2) + (\alpha + \beta)(\alpha - \beta) = 2(S^2 - 2P) + S\sqrt{S^2 - 4P}$

$\xrightarrow{S = \frac{-(-8)}{2} = 4} 2(16 - 2P) - 4\sqrt{16 - 4P} = 32 - 4P - 8\sqrt{4 - P} = 12 \Rightarrow 4P + 8\sqrt{4 - P} = 20$

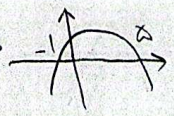
$\Rightarrow \quad 2\sqrt{4 - P} = 5 - P \Rightarrow 16 - 8P = P^2 - 10P + 25 \Rightarrow P^2 - 2P + 9 = (P - 3)^2 = 0 \Rightarrow P = 3$

$\Rightarrow \frac{m+2}{2} = 3 \Rightarrow m + 2 = 6 \Rightarrow m = 4$

۵

$y = ax^2 + bx + c \xrightarrow{(0,5)} c = 5 \Rightarrow y = ax^2 + bx + 5 \quad -\frac{b}{2a} = 2 \Rightarrow b = -4a$

$\Rightarrow ax^2 - 4ax + 5 = y \xrightarrow{(2,9)} 9 = 4a - 8a + 5 = 9 \Rightarrow -4a + 5 = 9 \Rightarrow a = -1$

$\Rightarrow y = -x^2 + 4x + 5 \xrightarrow{a+c=b} x = \begin{cases} -1 \\ 5 \end{cases} \Rightarrow$  $\Rightarrow y > 0 \rightarrow x \in (-1, 5)$

۶

$P = \frac{9\beta}{\alpha}$

$\alpha\beta^2 - 7\beta + 9\beta = \alpha\beta^2 + 2\beta = 0 \xrightarrow{\beta \neq 0} \alpha\beta + 2 = 0 \Rightarrow \alpha\beta = -2 = P$

$\Rightarrow \frac{9\beta}{\alpha} = -2 \Rightarrow 9\beta = -2\alpha \Rightarrow \alpha^2 - 7\alpha - 2\alpha = 0 \Rightarrow \alpha^2 - 9\alpha = \alpha(\alpha - 9) = \alpha(\alpha+2)(\alpha-3) = 0$

$\Rightarrow \alpha = \begin{cases} 0 \Rightarrow P = 0 \Rightarrow \beta = 0 \text{ قابل قبول} \\ -2 \Rightarrow P = 9 \Rightarrow \beta = -9 \Rightarrow \beta = \frac{-7}{4} \text{ قابل قبول} \\ 3 \Rightarrow P = 9 \Rightarrow \beta = 3 \Rightarrow \beta = \frac{3}{9} = \frac{1}{3} \end{cases} \Rightarrow \frac{1}{\alpha} + \frac{1}{\beta} = \frac{1}{3} + \frac{1}{-3} = \frac{2}{3} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}$

۷

$x^2 = -mx + 2m \Rightarrow \alpha^2 - m\beta = -m\alpha - m\beta + 2m = -m(\alpha+\beta) + 2m = 8$

$\Rightarrow -mS = 8 - 2m \Rightarrow -m(-m) = 8 - 2m \Rightarrow m^2 + 2m - 8 = 0 \Rightarrow (m-2)(m+4) = 0$

$\Rightarrow m = \begin{cases} 2 \Rightarrow x^2 + 2x - 4 = 0 \Rightarrow S = -2 \\ -4 \Rightarrow x^2 - 4x + 8 = 0 \quad \Delta < 0 \Rightarrow \text{غ ق ق} \end{cases}$

۸

$\frac{\Delta}{-4a} = 8 \Rightarrow \Delta = -32a \Rightarrow 16 - 4m\left(\frac{m}{2} + 7\right) = -32a \Rightarrow 16 - 2m^2 - 28m = -32a$

$\Rightarrow 2m^2 - 4m - 16 = 0 \Rightarrow m^2 - 2m - 8 = (m-4)(m+2) = 0$

$\Rightarrow m = \begin{cases} 4 \Rightarrow f(x) = 4x^2 + 4x + \underbrace{2+7}_{a} \Rightarrow \Delta < 0 \Rightarrow \text{غ ق ق} \\ -2 \Rightarrow f(x) = -2x^2 + 4x - \underbrace{1+7}_{4} \Rightarrow x = \frac{-4 \pm \sqrt{16 - 4(-2)(4)}}{-4} = \frac{-4 \pm \sqrt{48}}{-4} = 1 \pm 2 \end{cases}$

$\Rightarrow x = \begin{cases} 3 = k \\ -1 \end{cases}$

۹

$y = ax^2 + bx + c \xrightarrow{(0,4)} c = 4 \Rightarrow y = ax^2 + bx + 4 \quad \frac{-b}{2a} = 2 \Rightarrow b = -4a \Rightarrow y = ax^2 - 4ax + 4$

$\xrightarrow{(2,4)} 4 = 4a - 8a + 4 \Rightarrow -4a = 2 \Rightarrow a = -\frac{1}{2} \Rightarrow y = -\frac{1}{2}x^2 + 2x + 4$

$x = \frac{-2 \pm \sqrt{4 - (-\frac{1}{2})(4)}}{-1} = \frac{-2 \pm \sqrt{6}}{-1} = 2 \pm \sqrt{6} \Rightarrow \frac{1}{2+\sqrt{6}} + \frac{1}{2-\sqrt{6}} = \frac{2+\sqrt{6} + 2-\sqrt{6}}{(2+\sqrt{6})(2-\sqrt{6})}$

$\frac{4}{4-6} = \frac{4}{-2} = -2$

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$4 - (4a+4)2 + 2a^2 + 6a + 2 = 4 - 8a - 8 + 2a^2 + 6a + 2 = 2a^2 - 2a - 2 = 0$

$\xrightarrow{a+b+c=0} a = \begin{cases} 1 \\ -\frac{1}{2} \end{cases} \Rightarrow \begin{cases} x^2 - 8x + 12 = 0 \Rightarrow (x-6)(x-2) = 0 \Rightarrow x = \begin{cases} 2 \\ 6 \end{cases} \\ x^2 - \left(\frac{4}{2} + 4\right)x + 4\left(\frac{1}{2}\right) + 2 = x^2 - \frac{8}{2}x + 16 = 0 \\ \Rightarrow \Delta < 0 \rightarrow \text{ریشه ندارد} \Rightarrow \text{غ ق ق} \end{cases}$

$\Rightarrow x = 6$