

$$2. (a-1)x^2 + x + 3$$

$$x_1, 2, -\frac{b}{2a} = \frac{-1}{2a-2}, 2 \rightarrow 4a-4, -1 \rightarrow (a-\frac{3}{4})$$

$$y_1 = \frac{1}{4}x^2 + x + 3 \rightarrow x_1 + 6, -2 \xrightarrow{\text{طول}} (x, 6)$$

چون درجه‌های عبارت‌ها 2 و 1 است پس $f(x)$ حتماً $mx^2 - 5x + m$ دارد. $\rightarrow a < 0$ و $m < 0$ و موارد دور شده‌اند پس $\Delta > 0$

$$m < 0, 25 - 4m^2 > 0 \rightarrow m < 0, -\frac{5}{2} < m < \frac{5}{2} \xrightarrow{\text{اشتراک}} \boxed{-\frac{5}{2} < m < 0}$$

$$S_1 \alpha + \beta = \frac{3-\sqrt{5}}{3} + \frac{3+\sqrt{5}}{3} = \frac{6}{3} = 2$$

$$P_1 \alpha \cdot \beta = \left(\frac{3-\sqrt{5}}{3}\right) \cdot \left(\frac{3+\sqrt{5}}{3}\right) = \frac{9-5}{9} = \frac{4}{9} \rightarrow y_1 x^2 - 2x + \frac{4}{9}$$

$$\boxed{y_1 9x^2 - 18x + 4}$$

$$2x^2 - 8x + m + 2 = 0$$

$$\alpha + \beta = 4$$

$$\boxed{m=4}$$

$$\alpha = 1, \beta = 3$$

$$\alpha = 1 \rightarrow \alpha = \frac{1}{4}m$$

$$8\alpha = 2m$$

$$\alpha \cdot \beta = \frac{m}{2} + 1 \rightarrow 3 \cdot \frac{m}{2} + 1 = \frac{m}{2} + 2$$

$$8\alpha - m - 2 = m - 2$$

$$2\alpha^2 + \alpha^2 + \beta^2 = 12 \rightarrow 2\alpha^2 + 16 - m - 2 = 12 \rightarrow 2\alpha^2 = m - 2$$

$$y_1 a(x-2)^2 + 9 \xrightarrow{(0,5)} 5 = 4a + 9 \rightarrow a = -1 \rightarrow y_1 -x^2 - 4x + 9$$

$$y_1 -x^2 + 4x + 5 \rightarrow \begin{array}{c} -1 \quad 5 \\ | \quad | \\ -6 \quad +9 \end{array} \rightarrow \boxed{x \in (-1, 5)}$$

$$x^2 - 7x + 12 = 0 \rightarrow \alpha + \beta = 7 \rightarrow 3 + \beta = 7 \rightarrow \beta = 4$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha \cdot \beta} = \frac{7}{\alpha \cdot \beta} \rightarrow \alpha \cdot \beta = 12$$

$$\alpha + \beta = 7, \alpha \cdot \beta = 12$$

$$x^2 + mx - 2m = 0 \rightarrow \alpha^2 + m\alpha - 2m = 0$$

$$\alpha + \beta = m$$

$$\beta^2 + m\beta - 2m = 0$$

$$\alpha \cdot \beta = -2m$$

$$\alpha^2 - m\beta = 8 \rightarrow \alpha^2 + \beta^2 = 8 + 2m$$

$$m^2 + 2m - 8 = 0 \rightarrow m^2 + 4m - 2m - 8 = 0$$

$$m = 2, -4 \rightarrow \alpha + \beta = m = 2$$

$$f(x) = mx^2 + 4x + \frac{m}{2} + 7$$

$$\frac{-\Delta}{4a} = 8 \rightarrow \frac{(16 - 2m^2 - 28m)}{4m} = \frac{m^2 + 14m - 8}{2m} \rightarrow m = -2$$

$$K = 3 \rightarrow -2x^2 + 4x + 6$$

$$y = a(x-2)^2 + 6 \rightarrow 4 = 4a + 6 \rightarrow a = -\frac{1}{2}$$

$$y = -\frac{1}{2}x^2 - 2 + 2x + 6 \rightarrow y = -\frac{1}{2}x^2 + 2x + 4 \rightarrow S = 4, P = 8$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha \cdot \beta} = \frac{4}{-8} = -\frac{1}{2}$$

$$x^2 - (4a+4)x + (3a^2 + 6a + 3) = 0$$

$$x = 2$$

$$4 - 8a - 8 + 3a^2 + 6a + 3 = 0 \rightarrow 3a^2 - 2a - 1 = 0 \rightarrow a = 1$$

$$x = 2, 6 \rightarrow x^2 - 8x + 12$$