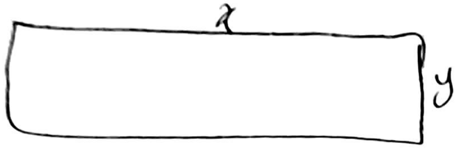
صدانم اصفهانی



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$$\frac{x}{y}=\frac{5}{4}\rightarrow x=\frac{5}{4}y$$

$$\frac{x+z}{y}=\frac{1+\sqrt{5}}{2}\qquad \frac{\frac{5}{4}y+z}{y}=\frac{1+\sqrt{5}}{2}$$

$$\frac{5}{2}y+2z=y+y\sqrt{5}\qquad \left(-\frac{3}{2}+\sqrt{5}\right)y=2z\rightarrow z=\frac{\left(\sqrt{5}-\frac{3}{2}\right)}{2}y$$

$$\frac{\text{ثانویه}}{\text{اولیه}}=\frac{(x+z)(\cancel{y})}{(x)(\cancel{y})}=\frac{\left(\frac{5-3+2\sqrt{5}}{4}\right)\cancel{y}}{\frac{5}{4}\cancel{y}}=\frac{\frac{1+\sqrt{5}}{\cancel{2}}}{\frac{5}{\cancel{4}\,2}}=\boxed{\frac{2+\sqrt{5}}{5}}$$

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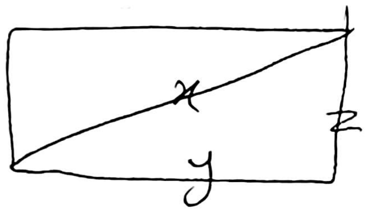

$$\frac{x}{y}=\frac{1+\sqrt{5}}{2}\rightarrow x=\frac{1+\sqrt{5}}{2}y$$

$$y^2+z^2=x^2\rightarrow y^2+z^2=\frac{6+2\sqrt{5}}{4}y^2\rightarrow z^2=\frac{2+2\sqrt{5}}{4}y^2$$

$$z^2=\frac{1+\sqrt{5}}{2}y^2$$

$$\left(\frac{y}{z}\right)^2=\frac{y^2}{z^2}=\frac{2}{1+\sqrt{5}}$$

۳-

$$3a+\sqrt{2a^2+4a}=2\qquad \frac{a+1}{a}$$

$$\sqrt{2a^2+4a}=2-3a\rightarrow 2a^2+4a=4+9a^2-12a\rightarrow 7a^2-16a+4=0$$

$$\Delta=256-112=144$$

$$\frac{16\pm12}{14}=\begin{matrix}2\ \times\\ \frac{2}{7}\ \checkmark\end{matrix}$$

$$\frac{\frac{2}{7}+1}{\frac{2}{7}}=\frac{\frac{9}{\cancel{7}}}{\frac{2}{\cancel{7}}}=\boxed{\frac{9}{2}}$$

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$$\frac{\sqrt{x+1}}{\sqrt{x-1}+3} - \frac{\sqrt{x+1}}{3-\sqrt{x-1}} = \frac{x-1}{\sqrt{x-1}} \qquad k=\sqrt{x-1}$$

$$\frac{\sqrt{k^2+2}}{k+3} - \frac{\sqrt{k^2+2}}{3-k} = \frac{\cancel{k^2}}{\cancel{k}} = k \qquad \frac{(3-k)\sqrt{k^2+2}-(k+3)\sqrt{k^2+2}}{9-k^2} = \frac{-2k\sqrt{k^2+2}}{9-k^2}$$

$$\frac{-2k\sqrt{k^2+2}}{9-k^2} = k \to \frac{-2\sqrt{k^2+2}}{9-k^2} = 1 \to -2\sqrt{k^2+2} = 9-k^2$$

$$2\sqrt{k^2+2} = k^2-9 \to 4(k^2+2) = (k^2-9)^2 \to k^4-22k^2+73=0$$

$$k^2=y \to y^2-22y+73=0 \to y=11\pm4\sqrt{3} \to t=\sqrt{11+4\sqrt{3}}$$

$$t^2+1=x \to \boxed{12+4\sqrt{3}=x}$$ فقط ۱ جواب مثبت

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$$\frac{1}{\sqrt{2-x}+2} - \frac{1}{2-\sqrt{2-x}} = \frac{2-x}{5\sqrt{2-x}} \qquad \sqrt{2-x}=k$$

$$\frac{1}{k+2} - \frac{1}{2-k} = \frac{\cancel{k^2}}{5\cancel{k}} \to \frac{+2k}{4-k^2} = \frac{k}{5}$$

$$10k = 4k-k^3 \to k^3+6k=0 \to k(k^2+6)=0$$

$k=0 \to \sqrt{2-x}=0 \to \boxed{x=2}$

$k=+\sqrt{6} \to \sqrt{2-x}=0 \to \boxed{x=-4}$

$k=-\sqrt{6} \to \sqrt{2-x}=-\sqrt{6}$ ✗

$\boxed{\text{فقط ۱ جواب}}$

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$$\frac{1}{x^2}+\frac{1}{(1-x)^2} = \frac{160}{9} \qquad \left(\frac{1}{x}+\frac{1}{1-x}\right)^2 - 2\left(\frac{1}{x(1-x)}\right) = \frac{160}{9}$$

$$\left(\frac{1}{x(1-x)}\right)^2 - 2\left(\frac{1}{x(1-x)}\right) = \frac{160}{9}$$

$$t^2-2t=\frac{160}{9} \to (t-1)^2 = \frac{169}{9} \to t-1=\pm\frac{13}{3}$$

$$t-1=\frac{13}{3} \to t=\frac{16}{3}$$

$$t-1=-\frac{13}{3} \to t=-\frac{10}{3} \to \frac{16}{3}-\frac{10}{3}=\frac{6}{3}=2$$

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$$\sqrt{x+\sqrt{\underbrace{-x^3+4x^2+25x-100}_{\geq 0}}} + \sqrt{x^2+\sqrt{\underbrace{-x^2+6x-8}_{\geq 0}}} = x+2$$

$$x^3-4x^2-25x+100 \leq 0 \to 4 \leq x \leq 5$$

$$(x-2)(x-4) \leq 0 \to 2 \leq x \leq 4$$

اشتراک $= x=4 \to \sqrt{4}+\sqrt{16}=6$ یک جواب $x=4$

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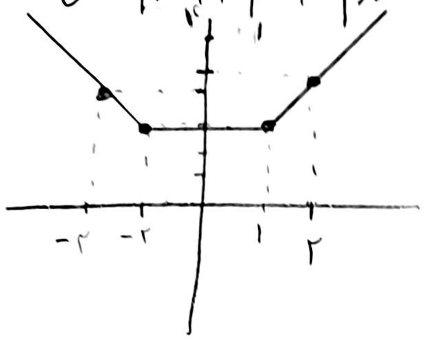
$y = |x+2| + |x-1|$



$3y + x = 17$

$y = -\frac{1}{3}x + \frac{17}{3}$

$-2x-1$	3	$2x+1$

$2x+1 = -\frac{1}{3}x + \frac{17}{3}$

$6x+3 = -x+17$

$7x = 14$

$x = 2$

$-2x-1 = -\frac{1}{3}x + \frac{17}{3}$

$-6x-3 = -x+17$

$5x = -20 \rightarrow x = -4$

$A\begin{vmatrix}-4\\+7\end{vmatrix} \quad B\begin{vmatrix}2\\5\end{vmatrix}$

$\sqrt{(2+4)^2 + (5-7)^2} = 2\sqrt{10}$

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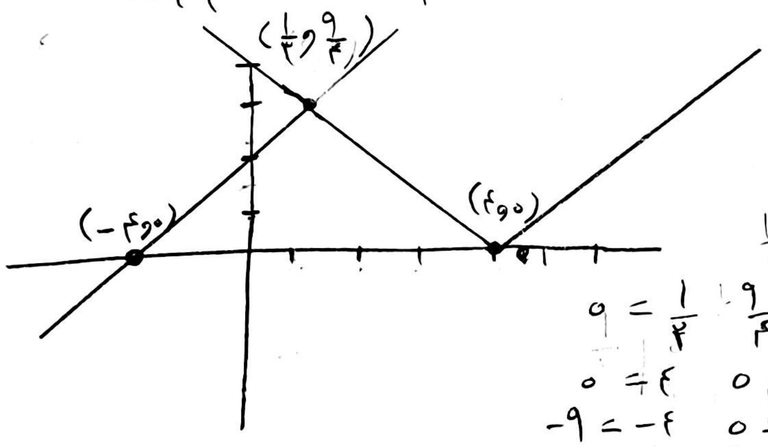
$y = \sqrt{x^2 - 4x + 4} \qquad y = \frac{1}{2}x + 2 \qquad x^2 - 4x + 4 = \frac{1}{2}x + 2$



$x^2 - \frac{9}{2}x + 2 = 0$

$\rightarrow x = 4$ ✗

$x = \frac{1}{2}$ ✓

$\frac{1}{4} + 2 = \frac{9}{4}$

$9 = \frac{1}{2} \quad \frac{9}{4} = 9$

$0 = 4 \quad 0 = 0$

$-9 = -4 \quad 0 = 0$

$\frac{1}{2} \quad \frac{9}{4}$

$\frac{9+9}{2} = \boxed{9}$

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$t_B = t_F - 9$

$\frac{1}{t_B} + \frac{1}{t_F} = \frac{1}{20} \qquad \frac{1}{t_F - 9} + \frac{1}{t_F} = \frac{1}{20}$

$\frac{2t_F - 9}{t_F^2 - 9t_F} = \frac{1}{20}$

$40t_F - 180 = t_F^2 - 9t_F$

$\boxed{t_B = 36}$

$t_F^2 - 49t_F + 180 = 0$

$\frac{49 \pm 41}{2} = 45$ ✓ $= 4$ ✗