

$$\frac{1+\sqrt{5}}{2} \leadsto \frac{1+\sqrt{5}}{2} = \frac{5+2}{4} \rightarrow \frac{1+\sqrt{5}}{2} = \frac{7}{4} = 1.75 (1+\sqrt{5})$$

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$$\begin{aligned} 2a + \sqrt{2a^2 + 4a} = 2 &\rightarrow \sqrt{2a^2 + 4a} = 2 - 2a \\ &\xrightarrow{\text{بهره لایه}} 2 - 2a \geq 0 \rightarrow a \leq \frac{2}{2} \\ \Rightarrow 2a^2 + 4a = (2 - 2a)^2 &\Rightarrow 2a^2 + 4a = 4a^2 - 8a + 4 \\ \Rightarrow 2a^2 - 12a + 4 = 0 &\rightarrow a = \frac{12 \pm \sqrt{144 - 32}}{4} = \frac{12 \pm \sqrt{112}}{4} \\ a = \frac{1}{2} = ? &\rightarrow \text{چون } 2 - 2a \geq 0 \rightarrow a = \frac{2}{2} = 1 \end{aligned}$$

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$$\begin{aligned} \text{طول: } L &\rightarrow \frac{\sqrt{L^2 + w^2}}{L} = \frac{1+\sqrt{5}}{2} \rightarrow \frac{L^2 + w^2}{L^2} = \frac{(1+\sqrt{5})^2}{4} \\ \text{عرض: } w &\rightarrow \frac{L^2 + w^2}{L^2} = \frac{(1+\sqrt{5})^2}{4} \\ \Rightarrow 1 + \left(\frac{w}{L}\right)^2 &= \frac{1+\sqrt{5}}{2} + 1 \rightarrow \left(\frac{w}{L}\right)^2 = \frac{1+\sqrt{5}}{2} - 1 \\ \rightarrow \frac{1}{\frac{1+\sqrt{5}}{2}} &= \frac{1+\sqrt{5}}{2} - 1 \rightarrow \frac{1+\sqrt{5}}{2} - 1 = \frac{\sqrt{5}-1}{2} \approx 0.61 \end{aligned}$$

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$$\begin{aligned} \frac{\sqrt{x+1}}{\sqrt{x-1} + 2} - \frac{\sqrt{x+1}}{2 - \sqrt{x-1}} &= \frac{x-1}{\sqrt{x-1}} = t \\ \Rightarrow (11 + 4\sqrt{3}) + 1 &= 12 + 4\sqrt{3} \\ \sqrt{x+1} \left(\frac{1}{t+2} - \frac{1}{2-t} \right) &= \sqrt{t^2 + 2} \times \frac{-2t}{4-t^2} = \frac{-2t\sqrt{t^2+2}}{4-t^2} = t \\ \text{فرض } y = \sqrt{t^2+2} &\Rightarrow y^2 = t^2 + 2 \Rightarrow t^2 = y^2 - 2 \\ \Rightarrow 2\sqrt{t^2+2} = t &\Rightarrow 2y = t \Rightarrow y = \frac{t}{2} \\ \Rightarrow y^2 = \frac{t^2}{4} &\Rightarrow \frac{t^2}{4} = y^2 - 2 \Rightarrow y^2 = \frac{t^2}{4} + 2 \end{aligned}$$

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$$\begin{aligned} \frac{1}{\sqrt{2-x} + 2} - \frac{1}{2 - \sqrt{2-x}} &= \frac{2-x}{2\sqrt{2-x}} \rightarrow \frac{2t}{t^2+2} = \frac{t}{2} \xrightarrow{t \neq 0} \frac{2}{t^2+2} = \frac{1}{2} \\ \Rightarrow 4 &= t^2 + 2 \Rightarrow t^2 = 2 \Rightarrow t = \pm\sqrt{2} \\ \Rightarrow x &= 2 - a^2 = 2 - 1 = 1 \end{aligned}$$

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جواب: ۵

$\frac{1}{a^r} + \frac{1}{(1-a)^r} = \frac{14}{9} \rightarrow \left(\frac{1}{a} + \frac{1}{1-a} \right)^r - r \left(\frac{1}{a(1-a)} \right) = \frac{14}{9}$ $\rightarrow \left(\frac{1}{a(1-a)} \right)^r - r \left(\frac{1}{a(1-a)} \right) = \frac{14}{9} \Rightarrow t^r - r t = \frac{14}{9}$ $\Rightarrow t^r - r t + 1 = \frac{14}{9} + 1 \Rightarrow (t-1)^r = \frac{14}{9} \Rightarrow t-1 = \pm \sqrt[r]{\frac{14}{9}}$ $\begin{cases} t-1 = \sqrt[r]{\frac{14}{9}} \rightarrow t = 1 + \sqrt[r]{\frac{14}{9}} \\ t-1 = -\sqrt[r]{\frac{14}{9}} \rightarrow t = 1 - \sqrt[r]{\frac{14}{9}} \end{cases} \rightarrow \frac{14}{9} - \frac{10}{9} = \frac{4}{9} = \boxed{2}$	6
$\sqrt{a} + \sqrt{-a^r + 4a^r + 10a - 1} + \sqrt{a^r + \sqrt{-a^r + 4a^r + 10a - 1}} = a + r$ $\Rightarrow a^r - 4a^r - 10a + 1 \leq 0 \rightarrow t \leq a \leq 0$ $\Rightarrow a(a-r)(a-t) \leq 0 \rightarrow r \leq a \leq t$ در جواب: $\boxed{a=t}$ $\rightarrow \sqrt{t} + \sqrt{14} = r+t = 9 \rightarrow a=t$ در جواب: $\boxed{a=t}$ $\rightarrow \sqrt{t} + \sqrt{14} = r+t = 9 \rightarrow a=t$	7
$y = a+r + a-1 \rightarrow \begin{matrix} \text{A} & \text{B} & \text{C} & \text{D} \\ \text{---} & \text{---} & \text{---} & \text{---} \end{matrix}$ $\sqrt{(r+t)^2 + (d-v)^2} = \sqrt{t^2} = r\sqrt{t}$ $\Rightarrow ①: (a+r) + (a-1) = 2a+1 \rightarrow r(2a+1) + a = w \rightarrow \forall a \geq 1 \rightarrow a \geq r$ $\Rightarrow ②: -r \leq a \leq 1 \rightarrow y = (a+r) + (1-a) = r+1 \rightarrow a=1 \rightarrow \text{در جواب: } \boxed{a=1}$ $\Rightarrow ③: a < -r \rightarrow y = -(a+r) + (1-a) = -2a-1 \rightarrow -r(-2a-1) + a = w \Rightarrow a = -r$	8
$y = \sqrt{a^r - 4a^r + 1} = a-r \xrightarrow{1^o} a \geq r \rightarrow a-r \rightarrow \frac{1}{r} a + r = a-r \rightarrow a = 1$ $y = \frac{1}{r} a + r \xrightarrow{1^o} a < r \rightarrow \frac{1}{r} a + r = r-a \rightarrow a = 0$ در جواب: $\frac{r+9}{r} \times 1 = t \times 1 = \boxed{12}$ $\rightarrow r=0 \rightarrow \frac{1}{r}(r)(r) = r \rightarrow 11+r = r$ $\rightarrow 12 \rightarrow \frac{1}{r}(4)(4) = 11 \Rightarrow \sqrt{12} = 12-r = \boxed{12}$	9
$\frac{1}{B} + \frac{1}{F} = \frac{1}{r} \Rightarrow \frac{1}{B} + \frac{1}{B+9} = \frac{1}{r} \rightarrow \frac{rB+9}{B(B+9)} = \frac{1}{r}$ $\Rightarrow r.B + 11 = B^2 + 9B \rightarrow B^2 - rB - 11 = 0$ $\rightarrow B = \frac{r \pm \sqrt{r^2 + 44}}{2}$ $\Rightarrow \frac{r+11}{2} = \boxed{12} \Rightarrow r = 11$	10