

۲۵ عالم ریاضت!

$$\frac{\sqrt{a}+1}{2} \leq \frac{a+1}{4} \Rightarrow 10+12 \leq 4\sqrt{a}+4 \Rightarrow 28 \leq 4\sqrt{a} \Rightarrow 7 \leq \sqrt{a} \Rightarrow a \geq 49$$

$$a+2\sqrt{a}-3 \leq 2+2\sqrt{a} \Rightarrow 5 \leq 2(1+\sqrt{a}) \Rightarrow 1+\sqrt{a} \geq \frac{5}{2} \Rightarrow \sqrt{a} \geq \frac{3}{2} \Rightarrow a \geq \frac{9}{4}$$

$$\frac{S_{\text{مربع}}}{S_{\text{مربع}}} = \frac{1+1\sqrt{a}}{2} \leq \frac{2+2\sqrt{a}}{4} \checkmark$$

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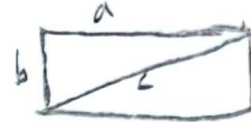
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$$\frac{\sqrt{a}+1}{2} \leq \frac{c}{a} \quad 2c \leq a\sqrt{a}+a$$

$$c \leq \frac{a\sqrt{a}+a}{2}$$

$$\frac{\sqrt{a^2+b^2}}{a} \leq \frac{\sqrt{a}+1}{2} \xrightarrow{2 \text{ طرف } \times} \frac{a^2+b^2}{a^2} \leq \frac{2+2\sqrt{a}}{2}$$

$$\Rightarrow 1+\frac{b^2}{a^2} \leq \frac{2+\sqrt{a}}{1} \quad \frac{b^2}{a^2} \leq \frac{1+\sqrt{a}}{1} \Rightarrow \frac{a^2}{b^2} \geq \frac{1}{1+\sqrt{a}} \checkmark$$



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$$\sqrt{a^2+9a} = 2-2a \Rightarrow a^2+9a = 4-16a \Rightarrow a^2+25a-4 = 0$$

$$\Rightarrow a^2+25a-4 = 0 \Rightarrow (a-1)(a-4) \Rightarrow a = \frac{1}{2} \checkmark \quad a = \frac{1}{2} = 2 \times 0.5 \quad 4+\sqrt{1+1} \neq 2$$

$$\frac{a+1}{a} \leq \frac{9}{2} \leq \frac{9}{2} \leq 4, 1 \checkmark$$

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$$\frac{\sqrt{x+1}}{\sqrt{x-1+2}} - \frac{\sqrt{x+1}}{2-\sqrt{x-1}} \leq \frac{x-1}{\sqrt{x+1}} \quad \sqrt{x-1} \leq t$$

$$\sqrt{x+1} \left(\frac{1}{t+2} - \frac{1}{2-t} \right) \leq \sqrt{t^2+2} \times \frac{-2t}{9-t^2} \Rightarrow \frac{-2t\sqrt{t^2+2}}{9-t^2} \leq t$$

$$2\sqrt{t^2+2} \leq t^2-9 \quad (y+2) \leq (y-9)^2 \quad 0 \leq t^2 \leq 9 \quad y^2-12y+13 \leq 0 \Rightarrow y = 11 \pm 4\sqrt{3} \rightarrow \boxed{\text{اربع شیب}} \checkmark$$

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$$\sqrt{2-x} \leq t \quad \frac{1}{t+2} - \frac{1}{2-t} \leq \frac{2-x}{4t} \Rightarrow 1+t-4t^2-1+t-4t^2 \leq 1+t^2$$

$$-6t^2 \leq 1-2t^2-4x+4t^2 \Rightarrow -1+t^2-1 \leq 4t^2-4x \xrightarrow{4t^2-4x}$$

$$-14+14x-1 \leq 14x-24 \Rightarrow x^2-10x-24 \leq 0 \Rightarrow (x+2)(x-12) \leq 0$$

$$x = -2, x = 12 \times 0.5$$

ریشه من است

ریشه مثبت ندارد

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$$9(x-1)^2 + 9x^2 = 14x^2(x-1)^2 \rightarrow 9x^2 - 18x + 9 + 9x^2 = 14x^2(x-1)^2$$

$$18x^2 - 18x + 9 = 14x^2 - 28x^2 + 14x^2 \rightarrow 14x^2 - 28x^2 + 14x^2 + 18x - 9 = 0$$

$$S = \frac{-b}{2a} = \frac{14}{14} = 1 \quad \checkmark$$

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$$-x^2 + 4x^2 + 20x - 100 \geq 0 \rightarrow x^2 - 4x^2 - 20x + 100 \leq 0 \Rightarrow 4 \leq x \leq 10 \quad (I)$$

$$x^2 - 4x + 4 \leq 0 \Rightarrow (x-2)(x-2) \leq 0 \quad 2 \leq x \leq 2 \quad (II)$$

$$I \cap II \Rightarrow x = 2 \quad \checkmark \quad \sqrt{4} + \sqrt{16} = 2 + 4 = 6$$

یک جواب دارد

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$$\sqrt{(x+1)^2 + (y-1)^2} = \sqrt{10} = 2\sqrt{10} \quad \checkmark \quad (x+1)(x-1) \leq 2(x+1) \rightarrow 2(x+1) + x \leq 14$$

$$4x + 2 + x = 14 \Rightarrow 5x = 12 \rightarrow x = 2.4 \rightarrow (2, 4)$$

$$-2 \leq x \leq 1 \Rightarrow y = (x+1) + (1-x) = 2 \rightarrow 2 + x = 14 \Rightarrow x = 12 \quad \times$$

$$x \leq -2 \Rightarrow y = -(x+1) + (1-x) = -2x \Rightarrow 2(-2x-1) + x \leq 14 \Rightarrow x \leq -4$$

$$y = -2(-4) = 8 \Rightarrow (-4, 8)$$

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$$[x-2] \rightarrow x \geq 2 \rightarrow \frac{1}{x} x + 2 = x - 2 \Rightarrow x = 4$$

$$x < 2 \rightarrow \frac{1}{x} x + 2 = 2 - x \Rightarrow x = 0$$

$$\frac{x+4}{x} \times 8 \leq 4 \times 8 \leq 32 \quad (I)$$

$$(2 \times 2 \times \frac{1}{x}) + (4 \times 4 \times \frac{1}{x}) \leq 2 + 16 \leq 18 \quad (II)$$

$$II - I \leq 32 - 2 \leq 12 \quad \checkmark$$

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$$\frac{1}{B} + \frac{1}{F} = \frac{1}{r} \Rightarrow \frac{1}{B} + \frac{1}{B+9} = \frac{1}{r} \Rightarrow \frac{r(B+9)}{B^2+9B} = \frac{1}{r}$$

$$\Rightarrow r(B+18) = B^2+9B \Rightarrow B^2 - rB - 18r = 0$$

$$B = \frac{r(1 \pm 5)}{2} \Rightarrow \frac{r(1+5)}{2} = 3r \quad \checkmark$$

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