

بار اولی

تکلیف ۱۸

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$$\frac{n}{y} = \frac{a}{r} \Rightarrow r n = a y \Rightarrow y = \frac{a}{r} n \Rightarrow n' = \frac{\sqrt{a} + 1}{r} y \Rightarrow n' = \frac{\sqrt{a} + 1}{r} \times \frac{a}{r} n = \frac{2\sqrt{a} + 2}{a} n$$

$$\Rightarrow \frac{a}{r} = \frac{n' y}{n y} = \frac{n'}{n} = \frac{\frac{2+2\sqrt{a}}{a} n}{n} = \frac{2+2\sqrt{a}}{a}$$

$$\frac{\sqrt{n^2 + y^2}}{n} = \frac{1+\sqrt{a}}{r} \Rightarrow \frac{n^2 + y^2}{n^2} = \frac{1+\sqrt{a}+2\sqrt{a}}{r} = \frac{3+2\sqrt{a}}{r} \Rightarrow 1 + \frac{y^2}{n^2} = \frac{3+2\sqrt{a}}{r} \Rightarrow \frac{y^2}{n^2} = \frac{2+2\sqrt{a}}{r}$$

$$\Rightarrow \left(\frac{n}{y}\right)^2 = \frac{n^2}{y^2} = \frac{r}{2+2\sqrt{a}} \times \frac{\sqrt{a}-1}{\sqrt{a}-1} = \frac{r(\sqrt{a}-1)}{r} = \frac{\sqrt{a}-1}{r}$$

$$\sqrt{2a^2 + 6a} = 2 - 2a \Rightarrow 2a^2 + 6a = 4a^2 - 12a + 4 \Rightarrow 2a^2 - 16a + 4 = 0 \Rightarrow a = \frac{16 \pm \sqrt{(16)^2 - (4 \times 2)}}{4}$$

$$\Rightarrow a = \frac{16 \pm \sqrt{256 - 8}}{4} = \frac{16 \pm 15}{4} = \frac{1 \pm 4}{1} = \frac{1}{4} < \frac{r}{r-4} \Rightarrow r-4 = -4 < 0 \Rightarrow \text{غیر ممکن}$$

$$\Rightarrow \frac{a+1}{a} = 1 + \frac{1}{a} = 1 + \frac{1}{\frac{1}{4}} = 1 + 4 = 5$$

$$\frac{\sqrt{n+1} - \sqrt{n^2-1} - \sqrt{n^2-1} - \sqrt{n^2-1}}{9 - (n-1)} = \sqrt{n-1} \Rightarrow \frac{-2\sqrt{n^2-1}}{10-n} = \sqrt{n-1} \Rightarrow \frac{-2\sqrt{n+1}}{1-n} = 1$$

$$\Rightarrow -2\sqrt{n+1} = 1-n \Rightarrow 2\sqrt{n+1} = n-1 \Rightarrow 4n+4 = n^2-2n+1 \Rightarrow n^2-6n-3=0$$

$$n = \frac{6 \pm \sqrt{36+12}}{2} = \frac{6 \pm \sqrt{48}}{2} = 3 \pm \sqrt{12} = \frac{12 \pm 4\sqrt{3}}{2} = \frac{12 \pm 4\sqrt{3}}{2} \Rightarrow \text{غیر ممکن}$$

یک ریشه مثبت دارد

$$\frac{2 - \sqrt{2-n} - 2 - \sqrt{2-n}}{2 - (2-n)} = \frac{-2\sqrt{2-n}}{n} = \frac{\sqrt{2-n}}{a} \Rightarrow \frac{-2}{n+2} = \frac{1}{a} \Rightarrow n+2 = -1 \Rightarrow n = -3$$

یک ریشه مثبت دارد

$$\Rightarrow 9(1-x)^2 + 9x^2 = 17 \cdot x^2(1-x)^2 \Rightarrow 9x^2 - 18x + 9 + 9x^2 = 18x^2 - 18x + 9 = 17x^2(1-x)^2$$

$$18x^2 - 18x + 9 = 17x^2 - 34x^3 + 17x^2 \Rightarrow 17x^2 - 34x^3 + 18x^2 - 18x + 9 = 0$$

$$x^3 + 2x^2 + 5x - 1 = 0$$

$$3y + n \leq 17 \Rightarrow 3y \leq -n + 17 \Rightarrow y \leq -\frac{1}{3}n + \frac{17}{3}$$

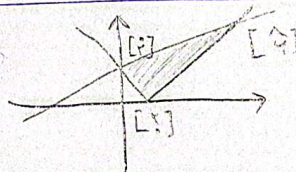
$$y \leq |x+2| + |x-1|$$

$$\begin{cases} -x-1 \leq -\frac{1}{3}n + \frac{17}{3} \Rightarrow -3x - 3 \leq -n + 17 \Rightarrow -3x \leq -n + 20 \Rightarrow x \geq \frac{n-20}{3} \\ 3 \leq -\frac{1}{3}n + \frac{17}{3} \Rightarrow 9 \leq -n + 17 \Rightarrow n \leq 8 \end{cases}$$

$$\Rightarrow AB = \sqrt{(x-(-1))^2 + (y-2)^2} = \sqrt{16 + 4} = \sqrt{20}$$

$$y = \sqrt{x^2 - 2x + 5} = \sqrt{(x-1)^2 + 4}$$

$$y = \frac{1}{2}x + 2$$



$$\Rightarrow |x-2| \leq \frac{1}{2}x + 2 \Rightarrow \begin{cases} x-2 \leq \frac{1}{2}x + 2 \Rightarrow \frac{x}{2} \leq 4 \Rightarrow x \leq 8 \\ x-2 \geq -\frac{1}{2}x - 2 \Rightarrow \frac{3x}{2} \geq 0 \Rightarrow x \geq 0 \end{cases}$$

$$\begin{vmatrix} 0 & 2 & 1 \\ 2 & 0 & 4 \\ 1 & 1 & 1 \end{vmatrix} \Rightarrow S = \frac{1}{2} |0 + 12 + 16 - (0 + 0 + 4)| = \frac{1}{2} \times 24 = 12$$

$$\frac{1}{n} + \frac{1}{n+9} \leq \frac{1}{2} \Rightarrow 2(n+9) + 2n \leq n^2 + 9n \Rightarrow n^2 - 4n - 18 \leq 0$$

$$(n-24)(n+9) \leq 0 \Rightarrow n \in \left[-9, \frac{24}{1} \right] \Rightarrow \text{SOL}$$