

۲۰ مثال نوشته

$$y = 3x^2 - ax + 4 = 0$$

ریشه ها α و 3α

$$S = 4\alpha = \frac{a}{3} \quad \begin{matrix} \alpha = \frac{1}{3} \rightarrow a = 1 \\ \alpha = -\frac{1}{3} \rightarrow a = -1 \end{matrix} \rightarrow 1 - (-1) = \boxed{14} \checkmark$$

$$P = 3\alpha^2 = \frac{4}{3} \rightarrow \alpha^2 = \frac{4}{9} \rightarrow \alpha = \pm \frac{2}{3}$$

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$$y = 34x^2 - (m+14)x + 1 = 0$$

$$P = \frac{1}{34}, S = \frac{m+14}{34}$$

ریشه ها $\sqrt{\frac{1}{\alpha}}$ و $\sqrt{\frac{1}{\beta}}$

$$\sqrt{\frac{1}{\alpha}} + \sqrt{\frac{1}{\beta}} = d$$

$$\frac{1}{\alpha} + \frac{1}{\beta} + 2\sqrt{\frac{1}{\alpha\beta}} = 2d \rightarrow \frac{\alpha+\beta}{\alpha\beta} = 13 \rightarrow \frac{m+14}{34} = m+14 = 13 \rightarrow \boxed{m = -1} \checkmark$$

$$y = -x^2 + 3x + 2 = 0$$

$$L \rightarrow P = \frac{c}{a} = \frac{2}{-1} = \boxed{-2} \checkmark$$

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$$y = 4x^3 + kx^2 - 9x - 2 = 0$$

$$\begin{matrix} \text{مقادیر} \\ \alpha = 2 \end{matrix} \rightarrow 32 + 4k - 18 - 2 = 0$$

$$\alpha/\beta = -2 \rightarrow \alpha = \frac{-2}{\beta}$$

$$4k = -12 \Rightarrow \boxed{k = -3} \checkmark$$

$$\alpha + \beta = 1 \rightarrow \frac{-2}{\beta} + \beta = 1$$

$$\xrightarrow{\alpha\beta} -2 + \beta^2 - \beta = 0 \rightarrow \beta^2 - \beta - 2 = 0 \rightarrow \begin{matrix} \beta = -1 \rightarrow \alpha = 2 \\ \beta = 2 \rightarrow \alpha = -1 \end{matrix}$$

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$$x^2 + 9x + a = 0 \rightarrow \begin{cases} \alpha = -9 + \sqrt{9-a} \rightarrow \alpha^2 = 18 - a - 4\sqrt{9-a} \\ \beta = -9 - \sqrt{9-a} \rightarrow \beta^2 = 18 - a + 4\sqrt{9-a} \end{cases}$$

$$3\alpha^2 + \beta^2 = 90 - 2a - 4\sqrt{9-a} = 12\sqrt{2} + 14 \rightarrow 2a + 4\sqrt{9-a} = 14 + \sqrt{14} \rightarrow \boxed{a = 1} \checkmark$$

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$$x^2 = t \Rightarrow t^2 - vt - d = 0 \rightarrow \Delta = 49 - 4(-d) = 49$$

$$L \rightarrow t = \frac{v \pm \sqrt{49}}{2} \rightarrow \begin{matrix} t_1 = \frac{v + \sqrt{49}}{2} \text{ ق ق} \\ t_2 = \frac{v - \sqrt{49}}{2} \text{ غ غ} \end{matrix}$$

$$x^2 = \frac{v + \sqrt{49}}{2} \rightarrow x_{1,2} = \pm \sqrt{\frac{v + \sqrt{49}}{2}}$$

$$L \rightarrow S = 0, P = \frac{v + \sqrt{49}}{2}$$

$$L \rightarrow 2P^2 = x \left(\frac{49 + 14\sqrt{49} + 49}{4} \right) = \frac{111 + 14\sqrt{49}}{2} = \boxed{49 + 7\sqrt{49}} \checkmark$$

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$y = 2ax^2 + ax - 4 = 0$ $\left(\begin{array}{l} \text{معمولی: } \alpha, \beta \\ S = \frac{-a}{2a} = -\frac{1}{2} \\ P = \frac{-c}{2a} = \frac{-3}{a} \end{array} \right.$ $S = \left[\frac{ab}{c} \right] = \left[\frac{1(-4)}{2} \right] = -2 \quad \checkmark$	$y = 2x^2 - ax + b = 0$ $\alpha + \frac{1}{\alpha}, \beta + \frac{1}{\beta} \rightarrow S = \alpha + \beta + 1 = -\frac{1}{2} + 1 = \frac{1}{2}$ $\frac{a}{2} = \frac{1}{2} \Rightarrow a = 1 \leftarrow \left(\frac{1}{2} \right)$ $P = (\alpha + \frac{1}{\alpha})(\beta + \frac{1}{\beta}) = \alpha\beta + \frac{1}{\alpha}(\alpha + \beta) + \frac{1}{\beta}$ $\alpha\beta = -\frac{3}{a}$ $\frac{b}{2} = \frac{-c}{2} = -3 \Rightarrow b = -6 \quad \checkmark$
$20\beta^2 + 20\alpha^2 + 20\beta^2 - 40\beta = 14$ $20[(\alpha + \beta)^2 - 2\alpha\beta] + 20(\beta^2 - \beta) = 14$ $\hookrightarrow \frac{b}{a} = -\frac{1}{10}$ $20(1 + \frac{2b}{a}) + 20(\frac{b}{a}) = 14$ $\rightarrow \frac{b}{a} = -\frac{1}{10}$	$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = 1 - 4 \times \frac{1}{20} = \frac{9}{10}$ $\alpha - \beta = \frac{3}{\sqrt{10}} \quad \checkmark$
$x^2 - (a+1)x + a = 0$ $\hookrightarrow n, n+2 \text{ (نز = n)}$ $P = n(n+2) = a \rightarrow P = 3$ $S = 2n+2 = a+1$ $2n = a-1 \rightarrow a = 2n+1$ $\rightarrow 2n+1 = n^2 + 2n \rightarrow n^2 = 1 \rightarrow n = 1 \Rightarrow a = 3 \quad \checkmark$	$y = x^2 - (3a+1)x + b = 0$ $\rightarrow S = 3a+1 \quad a=3, S=10 \xrightarrow{\text{ریشه}} 4, 6$ $\rightarrow P = b \rightarrow P = 4 \times 6 = 24 \quad \checkmark$ $24 - 3 = 21 \quad \checkmark$
$y = x^2 + x - 1 - m^2 = 0$ $\hookrightarrow S = \alpha + \beta = -1$ $\hookrightarrow P = \alpha\beta = -1 - m^2$ $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 1 - 2(-1 - m^2) = 3 + 2m^2$	$\boxed{3} \rightarrow \text{پاسخ}$ $\hookrightarrow m = 0$
$A(3, y), B(-1, y)$ $\hookrightarrow x_1 = \frac{3-d}{2} = -1 \rightarrow x_2 = (-1, 1)$ $y = a(x+1)^2 + 1 = ax^2 + 2ax + a + 1$ $\rightarrow S = \frac{-2a}{a} = -2, P = \frac{a+1}{a}$	$\alpha^2 + \beta^2 = S^2 - 2P = 4 - 2(\frac{a+1}{a}) = 4 - \frac{2a+2}{a}$ $\rightarrow \frac{4a - 2a - 2}{a} = \frac{2a-2}{a} = d$ $2a = 2a - 2 \rightarrow 2a = 2 \rightarrow a = 1 \quad \checkmark$ $\rightarrow y = -\frac{2}{3}x^2 - \frac{4}{3}x + \frac{1}{3}$ $x=0 \rightarrow y = \frac{1}{3} \Rightarrow (0, \frac{1}{3}) \quad \checkmark$