نام و نام خانوادگی ...[illegible]... پاسخنامه تشریحی تکلیف شماره ...... کلاس 17/5

1 ②

$\alpha \times 3\alpha = 3\alpha^2 = 4 \to \alpha^2 = \frac{4}{3} \to \alpha = \pm\frac{2}{\sqrt3}$

$-\frac{b}{a} = \alpha + 3\alpha = 4\alpha$ ($\frac{2}{\sqrt3} \to \frac{8}{\sqrt3} \to -\frac{a}{3} = \frac{8}{\sqrt3} \to a = 8\sqrt3$; $-\frac{2}{\sqrt3} \to -\frac{8}{\sqrt3} \to -\frac{a}{3} = -\frac{8}{\sqrt3} \to a = -8\sqrt3$)

$8\sqrt3 - -8\sqrt3 = +16\sqrt3$

2 ②

$\frac{1}{\sqrt\alpha}+\frac{1}{\sqrt\beta} = 5 \to \frac{1}{\alpha}+\frac{1}{\beta}+\frac{2}{\sqrt{\alpha\beta}} = 25$ $\alpha+\beta = \frac{m+14}{36}$ $\alpha\beta = \frac{1}{36}$

$\frac{\alpha+\beta}{\alpha\beta}+\frac{2}{\sqrt{\alpha\beta}} = 25 \to \frac{\frac{m+14}{36}}{\frac{1}{36}} + \frac{2}{\frac{1}{6}} = 25$ $m+14+12 = 25 \to m = -1$

$\frac{2}{m} = \frac{2}{-1} = -2$ ✓

3 ②

$(x-\alpha)^2 \times (x-\beta) \to (x^2+\alpha^2-2\alpha x)(x-\beta) = \cancel{x^3+\alpha^2 x - 2\alpha x^2}$

$x^3+\alpha^2 x - 2\alpha x^2 - \beta x^2 - \beta\alpha^2 - 2\alpha\beta x = \underbrace{1}_{4}x^3 - \underbrace{(2\alpha+\beta)}_{k}x^2 + \underbrace{(\alpha^2+2\alpha\beta)}_{-9}x - \underbrace{\beta\alpha^2}_{-2}$

$-\beta\alpha^2 = -\frac{2}{4} \to \beta\alpha^2 = \frac{1}{2}$ $\frac{\beta\alpha^2}{\alpha\beta} = \alpha = \frac{\frac12}{-2} = -\frac{1}{4}$

$2\alpha+\beta = \alpha+\beta+\alpha = 1-\frac{1}{4} = \frac{3}{4}$ $k = -(2\alpha+\beta)\times 4 = -\frac{3}{4}\times 4 = -3$ ✓

4 ②

$\alpha+\beta = -6 \to \beta = -6-\alpha \to \beta^2 = 36+\alpha^2+12\alpha \to 2\beta^2 = 72+2\alpha^2+24\alpha$

$3\alpha^2+72+2\alpha^2+24\alpha = 12\sqrt2+85$ $5\alpha^2+24\alpha-13+12\sqrt2 = 0$

$\alpha = \frac{-24\pm\sqrt{6+240\sqrt2}}{10}$ ($\frac{-9+10\sqrt5}{5}$ غ ق ق × ; $-3-2\sqrt2$) $-3-2\sqrt2-6 = \beta \to -3+2\sqrt2$ $-\alpha-6 = \beta$

$(-3+2\sqrt2)(-3-2\sqrt2) = 9-8 = 1$ ✓

5 ②

$x^4-7x^2-5 = 0 \to t^2-7t-5 = 0 \to t = \frac{7\pm\sqrt{(-7)^2-4(1)(-5)}}{2} = \frac{7\pm\sqrt{49+20}}{2} = \frac{7\pm\sqrt{69}}{2}$

$t>0 \to \frac{7-\sqrt{69}}{2}$ × $\to \frac{7+\sqrt{69}}{2}$ ✓ $x_1 = \sqrt{\frac{7+\sqrt{69}}{2}}$ و $x_2 = -\sqrt{\frac{7+\sqrt{69}}{2}}$

$S \to \sqrt{\frac{7+\sqrt{69}}{2}} - \sqrt{\frac{7+\sqrt{69}}{2}} = 0$ $P = x_1\times x_2 = -\left(\frac{7+\sqrt{69}}{2}\right)$

$2P^2-3SP+2S \xrightarrow{S=0} 2P^2 \to P^2 = \left(\frac{7+\sqrt{69}}{2}\right)^2 = \frac{(7+\sqrt{69})^2}{4} = \frac{49+14\sqrt{69}+69}{4} = \frac{118+14\sqrt{69}}{4}$

$2P^2 = 2\times\frac{118+14\sqrt{69}}{4} = \frac{118+14\sqrt{69}}{2} = 59+7\sqrt{69}$ ✓

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$\alpha+\beta=\frac{a}{2}$ ✓ $\alpha'+\beta'=\alpha+\frac{1}{2}+\beta+\frac{1}{2}=\alpha+\beta+1=\frac{a}{2}+1=\frac{-a}{2a}$ $a^2+2a=-a \Rightarrow a^2+3a=0 \begin{cases} a=0 \\ a=-3 \end{cases}$ $\left[\frac{(-a)(3)}{2}\right]\cdot\left[\frac{-9}{2}\right]=[-3,0]$ $\alpha\beta=\frac{b}{2}$ ✓ $\alpha'\cdot\beta'=(\frac{1}{2}+\alpha)(\frac{1}{2}+\beta)=\frac{1}{4}+(\beta+\alpha)\frac{1}{2}+\alpha\beta=\frac{1}{4}+\frac{a}{4}+\frac{b}{2}=\frac{-b}{2a}$ $\frac{1}{4}+\frac{a}{4}+\frac{b}{2}=-\frac{3}{a} \begin{cases} a=0 \Rightarrow \frac{1}{4}+0+\frac{b}{2}=\frac{0}{0} \\ a=-3 \Rightarrow \frac{1}{4}-\frac{3}{4}+\frac{b}{2}=1 \Rightarrow -\frac{1}{2}+\frac{b}{2}=1 \Rightarrow \frac{b}{2}=\frac{3}{2} \Rightarrow b=3 \end{cases}$	۱	
$\alpha+\beta=-\frac{-a}{a}=1$ $\alpha=1-\beta$ $\alpha^2=1+\beta^2-2\beta$ $2\beta^2+1+\beta^2-2\beta-\beta=\frac{17}{25} \Rightarrow 3\beta^2-3\beta+\frac{3}{25}=0 \Rightarrow \beta^2-\beta+\frac{1}{25}=0$ دقت! $\Delta=1-4(1)(-\frac{1}{25})=1+\frac{1}{5}=\frac{6}{5}$ $\beta=\frac{1\pm\sqrt{\frac{6}{5}}}{2}$ $\alpha=1-\frac{1\pm\sqrt{\frac{6}{5}}}{2}$ $\lvert\alpha-\beta\rvert=\left\lvert\frac{1-1\pm\sqrt{\frac{6}{5}}}{2}-\frac{1\pm\sqrt{\frac{6}{5}}}{2}\right\rvert=\left\lvert1-1\mp\sqrt{\frac{6}{5}}\right\rvert=\left\lvert\mp\sqrt{\frac{6}{5}}\right\rvert=\sqrt{\frac{6}{5}}$	۱	۷
$\alpha+\alpha+2=2\alpha+2=a+1 \Rightarrow 2\alpha+1=a$ $\alpha(\alpha+2)=\alpha^2+2\alpha=a$ $2\alpha+1=\alpha^2+2\alpha \Rightarrow 1=\alpha^2 \Rightarrow \alpha=\pm1 \Rightarrow \alpha=1$ طبق شرط $\Rightarrow 2(1)+1=3=a$ $\beta+\beta+2=2\beta+2=3a+1 \Rightarrow 3(3)+1=10 \Rightarrow 2\beta=8 \Rightarrow \beta=4$ $\beta+2=6$ $(\beta)(\beta+2)=4(6)=24$ $\alpha\times(\alpha+2)=1(3)=3$ $24-3=21$ ✓	۲	۸
$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=S^2-2P$ $S=-\frac{b}{a}=-\frac{1}{1}=-1$ $P=\frac{c}{a}=\frac{-1-m^2}{1}=-1-m^2$ $\Delta=b^2-4ac \Rightarrow 1^2-4(1)(-1-m^2)\geq0$ $5+4m^2\geq0 \Rightarrow +4m^2\geq-5 \Rightarrow m^2\geq-\frac{4}{5} \Rightarrow m^2\geq0$ $-1^2-2(-1-m^2)=1+2+2m^2=3+2m^2 \Rightarrow 3+2(0)=3$ ✓ $m^2_{min}=0$	۲	۹
$\frac{3+(-5)}{2}=-1=x_s$ ✓ $S\begin{vmatrix}-1\\1\end{vmatrix}$ $ax^2+bx+c \Rightarrow ax^2+2ax+c$ ✓ $\frac{-b}{2a}=-1 \Rightarrow b=2a$ $-1 \Rightarrow a-2a+c=1 \Rightarrow -a+c=1 \Rightarrow c=1+a$ ✓ $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=5=2^2-2(\frac{1+a}{a})=4-2-2a=5$ $-3a=3$ $\alpha\beta=\frac{c}{a}=\frac{1+a}{a}$ $c=1-\frac{3}{2}=\frac{1}{2}$ $\boxed{a=\frac{-2}{3}}$ $\boxed{c=\frac{1}{3}}$	۱٫۵	۱۰

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سوال ۱

$\alpha \cdot 3\alpha = 3\alpha^2 = \frac{4}{3} \rightarrow \alpha^2 = \frac{4}{9} \rightarrow \alpha = \pm\frac{2}{3}$

$-\frac{b}{a} = \alpha + 3\alpha = 4\alpha \begin{cases} \frac{2}{3} \rightarrow 4\left(\frac{2}{3}\right) = \frac{8}{3} = \frac{a}{3} \rightarrow a = 8 \\ -\frac{2}{3} \rightarrow 4\left(-\frac{2}{3}\right) = -\frac{8}{3} = \frac{a}{3} \rightarrow a = -8 \end{cases}$

$8 - -8 = 16$ ✓

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$$2x^2-ax+b=0 \to \begin{cases}\alpha\\ \beta\end{cases} \to S=\frac{a}{2} \quad P=\frac{b}{2}$$

$$-\frac{b}{a}$$

$$2ax^2+ax-6=0 \to \begin{cases}\alpha-\frac{1}{2}\\ \beta-\frac{1}{2}\end{cases} \to S=\frac{a}{2}-1=\frac{-a}{2a}=-\frac{1}{2} \to a=1$$

$$P=\frac{b}{2}=\frac{-6}{2a}=-3 \to b=-6$$

$$\left[\frac{ab}{4}\right]=\left[-\frac{3}{2}\right]=\boxed{-2}$$

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$$S=\alpha+\beta=1 \to \alpha=1-\beta$$

$$40\beta^2+20(1-\beta)^2-20\beta=17 \to 20\beta^2-20\beta+1=0$$

$$20\beta(\underbrace{\cancel{\beta-1}}_{-\alpha})=-1 \to -20\alpha\beta=-1 \to \alpha\beta=\frac{1}{20}=\frac{-b}{a} \to a=-20b$$

$$-20\cancel{b}x^2+20\cancel{b}x-\cancel{b}=0 \to |\alpha-\beta|=\frac{\sqrt{\Delta}}{|a|}=\boxed{\frac{2}{\sqrt{5}}}$$