نام و نام خانوادگی [illegible] ..... پاسخنامه تشریحی تکلیف شماره .... کلاس ....

۱

$3x^2 + ax + 4 = 0 \Rightarrow \alpha , \beta \Rightarrow \alpha = 3\beta \Rightarrow \alpha+\beta \Rightarrow 3\beta+\beta = 4\beta \Rightarrow 4\beta = -\frac{b}{a} \Rightarrow 4\beta = \frac{a}{3}$

$\alpha\beta \Rightarrow 3\beta(\beta) \Rightarrow 3\beta^2 = \frac{c}{a} \Rightarrow 3\beta^2 = \frac{4}{3} \Rightarrow \beta^2 = \frac{4}{9} \Rightarrow \beta = \frac{2}{3} , -\frac{2}{3}$

$\Rightarrow$

$$\beta \begin{cases} \frac{2}{3} \Rightarrow 4(\frac{2}{3}) \Rightarrow \frac{a}{3} \Rightarrow a = 8 \\ -\frac{2}{3} \Rightarrow 4(-\frac{2}{3}) \Rightarrow \frac{a}{3} \Rightarrow a = -8 \end{cases} \Rightarrow 8-(-8) = 16$$

۲

$36x^2 - (m+14)x + 1 = 0 \Rightarrow \frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = 5 \Rightarrow \frac{\sqrt{\alpha}+\sqrt{\beta}}{\sqrt{\alpha\beta}} = 5 \Rightarrow \alpha\beta = \frac{1}{36} \Rightarrow \sqrt{\alpha\beta} = \frac{1}{6} \Rightarrow \sqrt{\alpha}+\sqrt{\beta} = \frac{5}{6}$

$(\sqrt{\alpha}+\sqrt{\beta})^2 = \frac{25}{36} \Rightarrow \alpha+\beta+2\sqrt{\alpha\beta} = \frac{25}{36} \Rightarrow 2\sqrt{\alpha\beta} \Rightarrow \frac{2}{6} = \frac{1}{3} \Rightarrow \alpha+\beta = \frac{25}{36} - \frac{12}{36} \Rightarrow \alpha+\beta = \frac{13}{36}$

$\Rightarrow -\frac{b}{a} = \frac{13}{36} \Rightarrow \frac{m+14}{36} = \frac{13}{36} \Rightarrow m+14 = 13 \Rightarrow m = -1$

$\Rightarrow -x^2 + 3x + 2 = 0 \Rightarrow \boxed{\frac{c}{a} = -2}$

۳

$4x^3 + kx^2 - 9x - 2 = 0 \Rightarrow \alpha+\beta = 1 , \alpha\beta = -2 \Rightarrow \alpha\beta = -2 \Rightarrow \alpha = -\frac{2}{\beta} \Rightarrow -\frac{2}{\beta} + \beta = 1$

$\Rightarrow -2 + \beta^2 = \beta \Rightarrow \beta^2 - \beta - 2 = 0 \Rightarrow (\beta-2)(\beta+1) = 0 \quad \beta = 2 \; (\alpha = -1) \quad \beta = -1 \; (\alpha = 2)$ فرقی ندارد کدام را بگذاریم

$\Rightarrow \beta = 2 \Rightarrow 4(8) + 4(k) - 9(2) - 2 = 0 \Rightarrow 4k = -12 \Rightarrow \boxed{k = -3}$

۴

$x^2 + 6x + a = 0 \Rightarrow 3\alpha^2 + 2\beta^2 = 12\sqrt{2} + 85 \quad \alpha < \beta < 0$

$S = \alpha+\beta = -6 \quad P = \alpha\beta = a \Rightarrow 2(\alpha^2+\beta^2) + \alpha^2 = 12\sqrt{2} + 85 \quad \Delta = 36 - 4a$

$\alpha , \beta = -3 \pm \sqrt{9-a} \Rightarrow$ برای ساده‌تر شدن $12\sqrt{2}$ فرض می‌کنیم α و β به شکل $-3 \pm k\sqrt{2}$ باشد

$\Rightarrow 3(-3-k\sqrt{2})^2 + 2(-3+k\sqrt{2})^2 \Rightarrow 12\sqrt{2}+85 \Rightarrow$ بصورت ساده شده $\Rightarrow 10k^2 + 45 + 6k\sqrt{2} = 85 + 12\sqrt{2}$

$\Rightarrow$ باید رادیکال‌ها برابر باشند $\Rightarrow 6k\sqrt{2} = 12\sqrt{2} \Rightarrow k = 2 \Rightarrow a = (-3-2\sqrt{2})(-3+2\sqrt{2})$

$\Rightarrow 9 - 8 = \boxed{a = 1}$

۵

$x^4 - 7x^2 - 5 = 0 \Rightarrow y = x^2 \Rightarrow y^2 - 7y - 5 = 0 \Rightarrow y = \frac{7 \pm \sqrt{69}}{2} \Rightarrow$ فقط مثبت $= \frac{7+\sqrt{69}}{2}$

$\Rightarrow x = \pm\sqrt{\frac{7+\sqrt{69}}{2}} \Rightarrow S , P \Rightarrow S = (a) + (-a) = 0 \Rightarrow P(-a)^2 = (\frac{7+\sqrt{69}}{2})^2$

$\Rightarrow 2P^2 - 3SP + 2S \Rightarrow 2P^2 \Rightarrow 2(\frac{7+\sqrt{69}}{2})^2 \Rightarrow \boxed{59 + 7\sqrt{69}}$

① ②

$2x^2 + ax - 6 = 0 \ (\alpha , \beta)$ و $2x^2 - ax + b = 0 \ (\alpha + 0.5 , \beta + 0.5)$

$\Rightarrow \alpha+\beta = \frac{-a}{2} \Rightarrow \alpha+\beta = -\frac{1}{2} \Rightarrow \alpha + 0.5 + \beta + 0.5 \Rightarrow \alpha+\beta+1 \Rightarrow -\frac{1}{2}+1 = \frac{1}{2} \Rightarrow$ جمع ریشه‌های معادله دوم

جمع ریشه‌های معادله اول

$\Rightarrow \frac{a}{2} = \frac{1}{2} \Rightarrow \boxed{a=1} \Rightarrow 2x^2+x-6=0 \Rightarrow \frac{c}{a} = \alpha\beta = -3 \Rightarrow (\alpha+0.5)(\beta+0.5) = \alpha\beta + 0.5\alpha + 0.5\beta + \frac{25}{100}$ 6

$\Rightarrow -3 + 0.5\left(-\frac{1}{2}\right) + \frac{25}{100} = -3 \Rightarrow \frac{c}{a} = \frac{b}{2} = -3 \Rightarrow \boxed{b=-6}$

$\Rightarrow \frac{ab}{4} \Rightarrow \frac{-6}{4} = -1.5 \Rightarrow [-1.5] = -2$

$ax^2 - ax - b = 0 \Rightarrow 40\beta^2 + 20\alpha^2 + 20\beta = 17 \Rightarrow \alpha+\beta = 1 \Rightarrow \alpha = 1-\beta$

$\Rightarrow 40\beta^2 + 20(1-\beta^2) - 20\beta = 17 \Rightarrow 40\beta^2 + 20 - 40\beta + 20\beta^2 - 20\beta = 17 \Rightarrow 60\beta^2 - 60\beta + 20 = 17$

$\Rightarrow 60\beta^2 - 60\beta + 3 = 0 \Rightarrow 20\beta^2 - 20\beta + 1 = 0 \Rightarrow \Delta = \sqrt{320} = 8\sqrt{5} \Rightarrow \beta = \frac{1}{2} \pm \sqrt{5}$

$\alpha = \frac{1}{2} \pm \sqrt{5}$ (اگر کدام را انتخاب کنیم فرقی ندارد
فقط یکی منفی و دیگری مثبت) $\Rightarrow \alpha+\beta \Rightarrow \frac{1}{2}+\sqrt{5}+\frac{1}{2}-\sqrt{5}$ ✓ $\Rightarrow |\alpha-\beta| \Rightarrow 2\times\frac{\sqrt{5}}{5} = \boxed{\frac{2\sqrt{5}}{5}}$ 7

$x^2 - (a+1)x + a = 0 \Rightarrow \alpha , \beta \Rightarrow 2k-1 , 2k+1 \Rightarrow \alpha+\beta = (2k-1)(2k+1) \Rightarrow 4k = a+1 \Rightarrow a = 4k-1$

$(2k-1)(2k+1) = 4k^2-1 = a \Rightarrow 4k^2-1 = 4k-1 \Rightarrow 4k^2-4k = 0 \Rightarrow k(4k-4) = 0 \Rightarrow k=0 , k=1$ ✓

$\boxed{a=3} \Rightarrow x^2 - (2a+1)x + b = 0 \Rightarrow \alpha , \beta = 2m , 2m+2 \Rightarrow \alpha+\beta = 2m+2m+2 = 4m+2 = 2a+1$

$\Rightarrow a=3 \Rightarrow 4m+2 = 10 \Rightarrow m=2 \Rightarrow \alpha , \beta = 4 , 6 \Rightarrow 4\times 6 = 24 \Rightarrow |a-b| = |3-24| = 21$ 8

$x^2 + x - 1 - m^2 = 0 \Rightarrow x^2 + x - (1+m^2) = 0 \Rightarrow \alpha+\beta = -1 \quad \alpha\beta = -1-m^2$

$\Rightarrow (\alpha+\beta)^2 = 1 \Rightarrow \alpha^2+\beta^2+2\alpha\beta = 1 \Rightarrow \alpha^2+\beta^2-2-2m^2 = 1 \Rightarrow \alpha^2+\beta^2-2m^2 = 3$

$\Rightarrow 2m^2 \geq 0 \Rightarrow m = 0 \Rightarrow \boxed{\alpha^2+\beta^2 = 3}$ 9

$A(3,y) , B(-5,y) \Rightarrow$ عرض $= 1 \Rightarrow \alpha^2+\beta^2 = 5 \Rightarrow \frac{3+(-5)}{2} = -1 \Rightarrow \frac{\alpha+\beta}{2} = -1 \Rightarrow \alpha+\beta = -2$

$\alpha^2+\beta^2 = 5 \Rightarrow (\alpha+\beta)^2 = \alpha^2+\beta^2+2\alpha\beta = 4 \Rightarrow 2\alpha\beta = -1 \Rightarrow \alpha\beta = -\frac{1}{2} \Rightarrow y = a(x-\alpha)(x-\beta)$

$1 = y(-1) = a(-1-\alpha)(-1-\beta) \Rightarrow (-1-\alpha)(-1-\beta) = \left(\frac{\beta-\alpha}{2}\right)\left(\frac{\alpha-\beta}{2}\right) = -\frac{(\alpha-\beta)^2}{4}$ 10

$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta = 4+2 = 6 \Rightarrow 1 = -a\frac{6}{4} \Rightarrow a = -\frac{2}{3} \Rightarrow y(0) = \left(-\frac{2}{3}\right)\left(-\frac{1}{2}\right) = \frac{1}{3}$

$\Rightarrow \boxed{C\left(0 , \frac{1}{3}\right)}$