

$$3x^2 - ax + 8 = 0 \quad \text{ریشه ها: } \alpha, 3\alpha \rightarrow \frac{1}{\alpha} + \frac{1}{3\alpha} = \frac{a}{3} \rightarrow \frac{4}{3\alpha} = \frac{a}{3} \rightarrow a = \frac{4}{\alpha}$$

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$$\Rightarrow \text{ضرب ریشه ها} \Rightarrow \frac{8}{3} = 3\alpha^2 \Rightarrow \alpha^2 = \frac{8}{9} \Rightarrow \alpha = \pm \frac{\sqrt{24}}{3}$$

$$\Rightarrow \text{مجموع ریشه ها} \Rightarrow \frac{a}{3} = \frac{4}{3} + 3 \Rightarrow \frac{a}{3} = \frac{13}{3} \Rightarrow a = 13 \quad \text{یا} \quad \frac{a}{3} = -\frac{4}{3} - 3 = -\frac{13}{3} \Rightarrow a = -13$$

$$\text{خواسته سوال: } 13 - (-13) = 26 \quad \checkmark$$

$$3x^2 - (m+1)x + 1 = 0 \quad \text{ریشه ها: } \alpha, \beta \rightarrow \frac{1}{\alpha} + \frac{1}{\beta} = 1 \Rightarrow \frac{\alpha + \beta}{\alpha\beta} = 1 \Rightarrow \frac{m+1}{\alpha\beta} = 1 \Rightarrow \alpha\beta = m+1$$

$$\text{ضرب ریشه ها} = \frac{1}{3} \Rightarrow \frac{m+1}{3} = \frac{1}{3} \Rightarrow m+1 = 1 \Rightarrow m = 0$$

$$\text{مجموع ریشه ها} = \frac{m+1}{3} \Rightarrow \frac{m+1}{3} = \frac{1}{3} \Rightarrow m+1 = 1 \Rightarrow m = 0$$

$$m+1+12 = 2 \Rightarrow m = -11$$

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$$mx^2 + 2x + 2 = 0 \Rightarrow -x^2 + 3x + 2 = 0 \rightarrow \text{خواسته سوال} = \text{ضرب ریشه ها} = \frac{2}{-1} = -2 \quad \checkmark$$

$$x^2 + kx^2 - 9x - 2 = 0 \quad \alpha + \beta = 1 \quad \alpha \cdot \beta = -2$$

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$$\alpha + \beta = 1 \Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = 1 \Rightarrow \alpha^2 + \beta^2 + 2(-2) = 1 \Rightarrow \alpha^2 + \beta^2 = 5$$

$$\Rightarrow \alpha^2 + \beta^2 - 2\alpha\beta = 1 - 2(-2) = 5 \Rightarrow (\alpha - \beta)^2 = 9 \Rightarrow \alpha - \beta = \pm 3$$

$$\left. \begin{array}{l} \alpha + \beta = 1 \\ \alpha - \beta = 3 \end{array} \right\} \Rightarrow \alpha = 2, \beta = -1$$

$$\left. \begin{array}{l} \alpha + \beta = 1 \\ \alpha - \beta = -3 \end{array} \right\} \Rightarrow \alpha = -1, \beta = 2$$

دو ریشه دارد که یکی ۱- و دیگری ۲ است

$$x^2 + kx^2 - 9x - 2 = 0 \xrightarrow{x=1} -1 + k + 9 - 2 = 0 \Rightarrow k = -6$$

$$\xrightarrow{x=2} 4 + 4k - 18 - 2 = 0 \Rightarrow k = -3 \quad \checkmark$$

$$x^2 + 4x + a = 0 \quad \alpha < \beta < 0 \rightarrow \alpha^2 = -4\alpha - a$$

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$$2\alpha^2 + 2\beta^2 = 12\sqrt{2} + 18 \Rightarrow (-4\alpha - a) + (2\alpha^2 + 2\beta^2) = 12\sqrt{2} + 18 \Rightarrow \alpha = \frac{12\sqrt{2} + 13 + 5a}{-6}$$

$$\Rightarrow \alpha = \frac{13 + 5a}{-6} - 2\sqrt{2}$$

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$$\beta = \frac{13 + 5a}{-6} + 2\sqrt{2} \quad \leftarrow \text{برای } \beta \text{ این مورد } \sqrt{2} + \sqrt{2} \text{ باید باشد}$$

$$\Rightarrow \text{مجموع ریشه ها} \Rightarrow \alpha + \beta = -6 \Rightarrow \frac{13 + 5a}{-6} + 2\sqrt{2} + \frac{13 + 5a}{-6} - 2\sqrt{2} = -6 \Rightarrow \frac{26 + 10a}{-6} = -6$$

$$\Rightarrow 26 + 10a = 36 \Rightarrow a = 1 \quad \checkmark$$

$$x^2 - 7x^2 - 9 = 0 \Rightarrow -6x^2 - 9 = 0 \Rightarrow \Delta = b^2 - 4ac = 81 - 36 = 45 \Rightarrow \Delta > 0$$

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$$\Rightarrow S = \frac{-b}{a} = 7 > 0 \quad \text{حاصل ضرب ریشه ها} = \frac{c}{a} = -9 < 0 \rightarrow \oplus \text{ برای یک ریشه مثبت و یک ریشه منفی است} \Rightarrow x = \pm \sqrt{t_1}$$

$$\Rightarrow x_1 = \sqrt{t_1} \quad x_2 = -\sqrt{t_1} \quad \text{مجموع ریشه ها} \Rightarrow S = x_1 + x_2 = 0$$

$$t_1 = \text{مطلوبه}$$

$$\Rightarrow t_1 = \frac{b + \sqrt{\Delta}}{2a} = \frac{7 + \sqrt{45}}{2}$$

$$P = x_1 \cdot x_2 = (\sqrt{t_1})(-\sqrt{t_1}) = -t_1 = -\frac{7 + \sqrt{45}}{2}$$

$$2P^2 - 3SP + 2S = 2P^2 = 2\left(-\frac{7 + \sqrt{45}}{2}\right)^2 = 2\left(\frac{49 + 49 + 14\sqrt{45}}{4}\right) = 49 + 7\sqrt{45} \quad \checkmark$$

$$2x^2 - ax + b = 0 \rightarrow \begin{matrix} \text{ریشه} \\ \text{ها} \end{matrix} = \alpha + \frac{1}{\alpha}, \beta + \frac{1}{\beta} \Rightarrow S = \alpha + \beta + 1 = \frac{a}{\gamma} \quad P = \alpha\beta + \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\alpha\beta} = \frac{b}{\gamma} \quad (1, 1.75)$$

$$4ax^2 + a^2x - 4 = 0 \rightarrow \begin{matrix} \text{ریشه} \\ \text{ها} \end{matrix} = \alpha, \beta \Rightarrow S = \alpha + \beta = \frac{-a}{4a} = -\frac{1}{4} \quad P = \alpha\beta = \frac{-4}{4a} \Rightarrow \alpha\beta = -\frac{1}{a} \quad (1.75)$$

$$\frac{S}{\alpha + \beta + 1} = \frac{a}{\gamma} \Rightarrow \frac{-1}{4} + 1 = \frac{a}{\gamma} \Rightarrow \frac{3}{4} = \frac{a}{\gamma} \Rightarrow a = \frac{3\gamma}{4} \quad \checkmark \quad \text{خواسته سوال: } \begin{bmatrix} a & b \\ \gamma & \epsilon \end{bmatrix} = \begin{bmatrix} (-4) \times 1 \\ \epsilon \end{bmatrix} = \begin{bmatrix} -4 \\ \epsilon \end{bmatrix} = \begin{bmatrix} -4 \\ -1 \end{bmatrix}$$

$$\alpha\beta + \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\alpha\beta} = \frac{b}{\gamma} \Rightarrow -\frac{1}{a} - \frac{1}{\epsilon} + \frac{1}{\epsilon} = \frac{b}{\gamma} \Rightarrow b = -\frac{\epsilon}{\gamma} \quad \checkmark \quad \text{وقت!}$$

$$ax^2 - ax + b = 0 \quad S = \alpha + \beta = \frac{a}{a} = 1 \quad P = \alpha\beta = \frac{b}{a} \quad a\beta^2 - a\beta - b = 0 \Rightarrow \beta^2 - \beta = \frac{b}{a}$$

برای برابری تعداد β و β^2 ها این $\frac{b}{a}$ مناسب نیست

$$\epsilon_0\beta^2 + \gamma_0\alpha^2 - \gamma_0\beta = 17$$

$$\alpha + \beta = 1 \Rightarrow \beta = 1 - \alpha$$

$$\epsilon_0(1-\alpha)^2 + \gamma_0\alpha^2 - \gamma_0(1-\alpha) = 17 \Rightarrow \epsilon_0 - 2\epsilon_0\alpha + \epsilon_0\alpha^2 + \gamma_0\alpha^2 - \gamma_0 + \gamma_0\alpha = 17$$

$$\Rightarrow 4_0\alpha^2 - 4_0\alpha + 3 = 0 \Rightarrow 4_0\alpha^2 - 4_0\alpha + 1 = 0 \quad \text{به صورت همان معادله} \rightarrow ax^2 - ax - b = 0$$

$$|\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|} \Rightarrow 4_0\alpha^2 - 4_0\alpha + 1 = 0 \Rightarrow \Delta: (-4_0)^2 - 4(4_0)(1) = 32_0$$

$$\Rightarrow |\alpha - \beta| = \frac{\sqrt{32_0}}{4_0} = \frac{4\sqrt{2}}{4_0} = \frac{\sqrt{2}}{10} \quad \checkmark$$

$$x^2 - (a+1)x + a = 0 \Rightarrow \begin{matrix} \text{ریشه} \\ \text{ها} \end{matrix} = \alpha, \alpha+1 \quad \alpha^2 + 2\alpha = a \quad \alpha^2 + 2\alpha = a+1$$

$$\hookrightarrow \begin{matrix} \text{ریشه} \\ \text{ها} \end{matrix} = 1, 3 \quad a = 3 \leftarrow 2\alpha + 1 = a$$

$$x^2 - (2a+1)x + b = 0 \Rightarrow \begin{matrix} \text{ریشه} \\ \text{ها} \end{matrix} = \beta, \beta+2 \quad 2\beta + 2 = 2a + 1 \Rightarrow 2\beta + 2 = 2(3) + 1 \Rightarrow 2\beta + 2 = 7 \Rightarrow 2\beta = 5 \Rightarrow \beta = \frac{5}{2}$$

$$2\alpha + 1 = \alpha^2 + 2\alpha \Rightarrow \alpha^2 = 1 \Rightarrow \alpha = 1$$

$$\text{ضرب ریشه های} \Rightarrow 1 \times 3 = 3 \quad \text{ضرب ریشه های} \Rightarrow 4 \times \frac{5}{2} = 10 \quad \text{اختلاف: خواسته سوال} \Rightarrow 2\epsilon - 3 = 21 \quad \checkmark$$

$$S = -\frac{b}{a} \quad P = \frac{c}{a}$$

$$x^2 + x - 1 - m^2 = 0 \rightarrow \begin{matrix} \text{ریشه} \\ \text{ها} \end{matrix} = \alpha, \beta \quad \alpha^2 + \beta^2 = S^2 - 2P = (1+m^2)^2 - 2 \Rightarrow (1)^2 - 2 = -1$$

$$\Delta = 1 + 4 + 4m^2 > 0 \Rightarrow \Delta \text{ همواره} \quad \text{مداخل باید} \quad \alpha^2 + \beta^2 = S^2 - 2P = (-1)^2 - 2(-1-m^2) = 2m^2 + 3 \geq 3 \quad \text{min}$$

$$\text{عرض} = 1 \quad \text{طول} \Rightarrow (-5, y), (3, y) \Rightarrow \text{مختصات} = \frac{y-5}{2} = -1$$

$$\text{رأس} \Rightarrow (-1, 1) \quad \text{نقطه} \Rightarrow (-1, -\sqrt{\frac{3}{2}})$$

$$\alpha \Rightarrow -1 - x \quad \beta \Rightarrow -1 + x \quad \alpha^2 + \beta^2 = 0$$

$$\Rightarrow x^2 + 2x + 1 + x^2 - 2x + 1 = 0 \Rightarrow 2x^2 + 2 = 0 \Rightarrow x^2 = -1 \Rightarrow x = \pm i$$

$$\Rightarrow 2x^2 = 3 \Rightarrow x^2 = \frac{3}{2} \Rightarrow x = \pm \sqrt{\frac{3}{2}}$$

$$y = a(x + 1 + \sqrt{\frac{3}{2}})(x + 1 - \sqrt{\frac{3}{2}}) \xrightarrow{(-1, 1)} 1 = a(-1 + 1 + \sqrt{\frac{3}{2}})(-1 + 1 - \sqrt{\frac{3}{2}})$$

$$\Rightarrow 1 = a(\sqrt{\frac{3}{2}})(-\sqrt{\frac{3}{2}}) \Rightarrow 1 = -\frac{3}{2}a \Rightarrow \frac{3}{2}a = -1 \Rightarrow a = -\frac{2}{3}$$

$$y = -\frac{2}{3}(x + 1 + \sqrt{\frac{3}{2}})(x + 1 - \sqrt{\frac{3}{2}}) \Rightarrow y = -\frac{2}{3}x^2 - \frac{4}{3}x + \frac{1}{3}$$

$$x^2 + 2x - \frac{1}{3}$$

در عرض $\Rightarrow \frac{1}{3}$

مختصات $(0, \frac{1}{3})$