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باز هم سر

$$2u^2 + y^2 - 2u + 2y + k = 0 \rightarrow 2u^2 - 2u + y^2 + 2y + k = 0$$

$$\rightarrow 2(u^2 - u) = 2((u-1)^2 - 1) \rightarrow 2(u-1)^2 - 2$$

$$y^2 + 2y = (y+1)^2 - 1 \rightarrow 2(u-1)^2 - 2 + (y+1)^2 - 1 + k = 0$$

$$2(u-1)^2 + (y+1)^2 + k - 3 = 0 \quad k - 11 < 0 \rightarrow k < 11$$

تابع نقطه (۱، ۳)

فقط برای $k=11$ تابع می شود
برای مقادیر کمتر k دو مرکز

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$$f(u) = \begin{cases} u^2 + \varepsilon u & u \geq a \\ 2u + a + 2 & u \leq a \end{cases}$$

$$f(u) = ?$$

$$\rightarrow a^2 + \varepsilon a = 2a + a + 2 \rightarrow a^2 + a - 2 = 0 \rightarrow (a-1)(a+2) = 0 \rightarrow \begin{cases} a=1 \\ a=-2 \end{cases}$$

$$a=1 \rightarrow 1^2 + \varepsilon(1) = 0 = 2(1) + 1 + 2 = 5$$

$$a=-2 \rightarrow \cancel{(-2)^2 + \varepsilon(-2) = 0 = 2(-2) + (-2) + 2 = -2} \rightarrow 1^2 + \varepsilon(u) = 0 \quad \checkmark$$

$$f(u) = \begin{cases} 2u^2 - 6 & |u+1| \geq 2 \\ a + 2u & -3 \leq u \leq -1 \end{cases} \quad a+b=?$$

$$|u+1| \geq 2 \rightarrow u+1 \geq 2 \rightarrow u \geq 1 \rightarrow -3 \leq u \leq -1$$

$$u=-1 \rightarrow 2(-1)^2 - 6 = a + 2(-1) \rightarrow 2 - 6 = a - 2 \rightarrow a+b = 8$$

$$u=-3 \rightarrow 18 - 6 = a - 6 \rightarrow a+b = 24$$

$$y = \sqrt{4u-a} + \sqrt{b-2u}$$

$$\begin{cases} 4u \geq a \\ u \geq \frac{a}{4} \end{cases}$$

$$\begin{cases} b \geq 2u \\ \frac{b}{2} \geq u \end{cases}$$

$$\rightarrow [\frac{a}{4}, \frac{b}{2}] \rightarrow \frac{b}{2} = \frac{a}{4} = 2 \rightarrow a=4, b=4$$

$$\frac{b}{a} = \frac{4}{4} = 1 \quad \checkmark$$

$$y = \sqrt{\frac{u+1}{u-1}} + \sqrt{\frac{4-u}{u+1}} \rightarrow u \geq -1, |u-1| > 0 \rightarrow u > 1 \rightarrow [-1, 3] \cap \{u < 1\} \rightarrow \emptyset$$

$$5-u \geq 0 \rightarrow u \leq 5 \rightarrow \frac{1}{5} = \frac{5-u}{u} \rightarrow \frac{1}{5} = \frac{5}{u} - 1 \rightarrow \frac{1}{5} + 1 = \frac{5}{u} \rightarrow \frac{6}{5} = \frac{5}{u} \rightarrow u = \frac{25}{6}$$

$$y = \sqrt{u-1} + \sqrt{1-u}$$

$$[u] \geq 1 \rightarrow [1, +\infty)$$

$$2 \geq [u] \rightarrow (-\infty, 2]$$

$$\phi$$

$$(-1, 3) \cap [-1, 5] = [-1, 3]$$

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المجموعة

$$y = \sqrt{\frac{u^2 - 9}{u^2 + 4u + 4}} \rightarrow \frac{(u-3)(u+3)}{(u^2-9)(u^2+4u+4)} \rightarrow \frac{-1}{+1} \rightarrow (-\infty, -3) \cup (3, +\infty)$$

$$y = \sqrt{\frac{u^2 - 4u + 4}{u^2 + 4u + 4}} \geq 0 \rightarrow \frac{(u-1)(u-3)(u+3)}{(u+1)(u+3)(u-3)} \rightarrow \frac{(u-1)}{(u+1)}$$

$\Delta = 1 - 9(1)(-4) = 37 > 0$

$\frac{-1}{2} \pm \frac{\sqrt{37}}{2}$

$$a) y = \sqrt{-u-2} \sim [-u] \geq 2 \rightarrow (-\infty, -2]$$

$$b) y = \frac{u^2 + 5}{[u]^2 - 2[u] - 3} \rightarrow ([u]-3)([u]+1) \neq 0$$

$[u] \neq 3, [u] \neq -1 \rightarrow [0, +\infty) \setminus \{3, -1\}$

$u \notin [3, 4), u \notin [-1, 0)$

$\mathbb{R} - \{[3, 4), [-1, 0)\}$

$$a) y = \frac{\cos u + 1}{\tan u + 1} \rightarrow \tan u \neq -1$$

$\sin u, \cos u \neq 0$

$b) y = \sqrt{1 - \sin^2 u}$

$1 - \sin^2 u \geq 0 \rightarrow 1 \geq \sin^2 u \rightarrow \frac{1}{2} \geq \sin u \rightarrow \frac{\sqrt{2}}{2} \geq \sin u \leq \frac{1}{2}$

$D = \mathbb{R} - \{K\pi - \frac{\pi}{4}, K\pi - \frac{\pi}{2}\}$

$[K\pi - \frac{\pi}{2}, K\pi + \frac{\pi}{2}] \cup [K\pi + \frac{\pi}{2}, K\pi + \frac{3\pi}{2}]$

$$a) y = \sqrt{1 - \log \frac{u-1}{u}} \rightarrow 1 \geq \log \frac{u-1}{u} \rightarrow \frac{1}{e} \leq \frac{u-1}{u} \rightarrow 1 - \frac{1}{u} \leq \frac{1}{e} \rightarrow u \leq \frac{e}{1-\frac{1}{e}} = \frac{e^2}{e-1}$$

$b) y = \sqrt{\frac{u^2}{1 - \log u - u}}$

$u^2 - u > 0 \rightarrow u(u-1) > 0 \rightarrow u < 0 \text{ or } u > 1$

$1 - \log u - u > 0 \rightarrow 1 - \log u > u$

$D = (\frac{e^2}{e-1}, +\infty) \cup (-\infty, 0)$

$$a) y = \sqrt{u^2 - u} \rightarrow u^2 - u \geq 0 \rightarrow u(u-1) \geq 0 \rightarrow u \leq 0 \text{ or } u \geq 1$$

$b) y = \left(\frac{u+d}{u+e}\right)!$

$\frac{u+d}{u+e} \in \mathbb{N} \rightarrow u+d = k(u+e) \rightarrow u - ku = ku + e - d \rightarrow u(1-k) = k(u+e) - d$

$u(1-k) = k(u+e) - d \rightarrow u = \frac{k(u+e) - d}{1-k}$

$\{u \mid u = \frac{k(u+e) - d}{1-k}, k \in \mathbb{N}\}$