

$$(10) \quad y = \frac{\sqrt{4x^2 - 4x + 1}}{\sqrt{4x^2 - 4x + 1}} \Rightarrow \frac{1}{1-1} \Rightarrow (-\infty, 1) \cup [1, +\infty)$$

$$\Rightarrow \frac{1}{1-\frac{1}{x}} \Rightarrow (-\infty, 1) \cup (\frac{1}{x}, +\infty)$$

$$\Rightarrow (-\infty, 1) \cup (\frac{1}{x}, +\infty) \Rightarrow D_f = (-\infty, 1) \cup (\frac{1}{x}, +\infty)$$

$$(11) \quad y = \frac{\sqrt{4x^2 - 4x + 1}}{\sqrt{4x^2 - 4x + 1}} \Rightarrow \frac{1}{1-\frac{1}{x}} \Rightarrow D_f = \mathbb{R} - [\frac{1}{x}, 1, \infty) \cup \{1\}$$

تکلیف دوم (04/05/1401) اسیرجی - نازق زاده

$$(1) \quad 2x^2 + y^2 - 4x + 4y + k = 0 \Rightarrow 2x^2 - 4x + 2 + y^2 + 4y + 4 - 2 - 4 + k = 0$$

$$\Rightarrow 2(x^2 - 2x + 1) + (y^2 + 4y + 4) + k - 11 = 0$$

$$\Rightarrow 2(x-1)^2 + (y+2)^2 + k - 11 = 0 \Rightarrow x+y \geq 11 \Rightarrow \text{تبعی نیست}$$

$$\Rightarrow k = 11 \quad \checkmark$$

$$(2) \quad f(x) = \begin{cases} x^2 + 4x; & x \geq 1 \\ 2x + a + 2; & x < 1 \end{cases} \Rightarrow f(a) = a^2 + 4a = 2a + a + 2 \Rightarrow a^2 + a - 2 = 0 \Rightarrow (a+2)(a-1) = 0 \Rightarrow a = -2 \text{ or } a = 1$$

$$\Rightarrow f(1) = 2(1)^2 + f(1) = 1 + 1 = 2 \Rightarrow f(1) = 2 \quad \checkmark$$

$$(3) \quad f(x) = \begin{cases} x^2 - b; & |x+1| \geq 2 \\ a + 2x; & -2 \leq x \leq -1 \end{cases} \Rightarrow \begin{cases} x+1 \geq 2 \Rightarrow x \geq 1 \\ x+1 \leq -2 \Rightarrow x \leq -3 \end{cases}$$

$$\Rightarrow f(-3) = 2 \times 9 - b = 2a + 2(-3) \Rightarrow 18 - b = 2a - 6 \Rightarrow a + b = 12 - 2b \quad \checkmark$$

$$(4) \quad y = \sqrt{4x-a} + \sqrt{b-2x} \Rightarrow \begin{cases} 4x-a \geq 0 \Rightarrow x \geq \frac{a}{4} \\ b-2x \geq 0 \Rightarrow x \leq \frac{b}{2} \end{cases}$$

$$\frac{b}{2} \geq \frac{a}{4} \Rightarrow \frac{a}{4} \geq \frac{a}{4} \Rightarrow a \geq 1, b \geq 4$$

$$\Rightarrow \frac{b}{2} \geq \frac{a}{4} \Rightarrow \frac{1}{2} \quad \checkmark$$

⑤ (الف) $\sqrt{\frac{x+1}{x-1}} + \sqrt{\frac{4-x}{x^2+2x+1}} =$ (2)

$\frac{x+1}{x-1} \geq 0 \Rightarrow \begin{cases} x \geq 1 \\ x \neq 1 \end{cases} \quad \frac{4-x}{x^2+2x+1} \geq 0 \Rightarrow \begin{cases} x \leq 4 \\ x \neq -1 \end{cases} \Rightarrow \textcircled{1}, \textcircled{2} \Rightarrow [-1, 4] - \{1\} = D_f$

① ②

→ $\sqrt{[x]-1} + \sqrt{4-[x]} =$

$[x]-1 \geq 0 \Rightarrow [x] \geq 1 \Rightarrow x \in [1, +\infty) \textcircled{1} \Rightarrow \textcircled{1} \cap \textcircled{2} = \emptyset \Rightarrow D_f = \emptyset$

$4-[x] \geq 0 \Rightarrow [x] \leq 4 \Rightarrow x \in (-\infty, 4) \textcircled{2}$

⑥ (ب) $\sqrt{\frac{x^2-x-4}{x^2-13x+14}} \Rightarrow \frac{x^2-x-4}{x^2-13x+14} \geq 0$

$\Rightarrow x^2-x-4=0 \Rightarrow \Delta = 1+16=17 \Rightarrow x_{1,2} = \frac{1 \pm \sqrt{17}}{2} \begin{cases} r^* \\ -r^* \end{cases}$

$x^2-13x+14=0 \Rightarrow t^2-13t+14=0 \Rightarrow \Delta = 169-196=25 \Rightarrow t_{1,2} = \frac{13 \pm 5}{2} \begin{cases} 9 \\ 4 \end{cases}$
($t=x^2$)

$\Rightarrow t_1 = x_1^2 = 9 \Rightarrow x_{1,2} = \pm 3$

$t_2 = x_2^2 = 4 \Rightarrow x_{3,4} = \pm 2$

$\frac{-r^*}{+} \frac{-r^*}{-} \frac{-r^*}{-} \frac{r^*}{+}$

$\Rightarrow D_f = (-\infty, 2) \cup (2, 3) \cup (3, +\infty)$

⑦ (ج) $\sqrt{\frac{x^2-2x^2-\Delta x+4}{x^2+2x^2-\Delta x-4}} \Rightarrow \frac{x^2-2x^2-\Delta x+4}{x^2+2x^2-\Delta x-4} \geq 0 \Rightarrow$ ①

$x^2-2x^2-\Delta x+4 = (x-2)(x+1)(x-2) \Rightarrow \omega_{x-2}, \begin{cases} r^* \\ r^* \\ -1^* \end{cases}$

$x^2-2x^2-\Delta x-4 = (x+3)(x+1)(x-2) \Rightarrow \omega_{x-2}, \begin{cases} -r^* \\ -1^* \\ r^* \end{cases}$

$\frac{-r^*}{+} \frac{-1^*}{-} \frac{r^*}{-} \frac{r^*}{+}$

$\Rightarrow D_f = (-\infty, -2) \cup [2, +\infty)$

$\begin{array}{cccccc} -2 & -2 & -1 & 1 & 2 & 2 \\ + & - & + & - & + & - \end{array}$

⑦ ا) $y = \sqrt{[-x] - 2} \Rightarrow [-x] - 2 \geq 0 \Rightarrow D_f = (-\infty, -1) \cup (-\infty, -2]$

ب) $y = \frac{\Delta x^2 + 4}{[x]^2 - 2[x] - 2} \Rightarrow [x]^2 - 2[x] - 2 \neq 0$ (1)

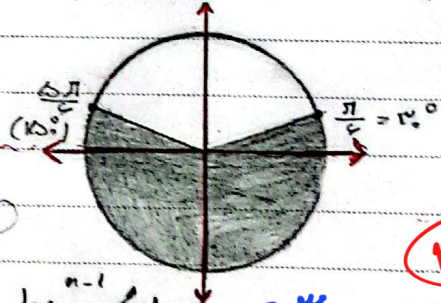
$\Rightarrow x^2 - 2x - 2 = 0 \Rightarrow \Delta = 4 + 16 = 20 \Rightarrow x_1, x_2 = \frac{2 \pm \sqrt{20}}{2} = 1 \pm \sqrt{5}$

$\Rightarrow [x_1], [x_2] = (1, 3), (0, -1) \Rightarrow D_f = \mathbb{R} - ([1, 3] \cup [0, -1])$ ✓

⑧ $\frac{\cot x + 1}{\tan x + 1} \Rightarrow \tan x + 1 > 0 \Rightarrow \tan x > -1 \Rightarrow \tan x \neq -1$ $\sin x, \cos x \neq 0$
 $\tan(135^\circ), \tan(315^\circ) = -1 \Rightarrow D_f = \mathbb{R} - \{k\pi - \frac{\pi}{4}\} \mathbb{R} - \{k\frac{\pi}{2}, k\pi - \frac{\pi}{4}\}$

ب) $y = \sqrt{1 - f \sin^2 x} \Rightarrow 1 - f \sin^2 x \geq 0 \Rightarrow f \sin^2 x \leq 1 \Rightarrow f \sin^2 x \leq 1$ (2, 2)

$\Rightarrow \sin^2 x \leq \frac{1}{f} \Rightarrow D_f = \{k\pi - \frac{\Delta \pi}{4}\}$
 $\sin x \neq \pm 1 \Rightarrow D = [k\pi - \frac{\pi}{4}, k\pi + \frac{\pi}{4}]$



⑨ $\sqrt{1 - \log \frac{n-1}{t}} \Rightarrow \begin{cases} 1 - \log \frac{n-1}{t} \geq 0 \Rightarrow \log \frac{n-1}{t} \leq 1 \Rightarrow \frac{n-1}{t} \leq 10 \Rightarrow n \leq 10t + 1 \end{cases}$ (1, 2)
 $\Rightarrow n-1 \leq \frac{1}{t} \Rightarrow n \leq 1 + \frac{1}{t} \Rightarrow D_f = (1, 1.5]$

ب) $\frac{\sqrt{x^2 - n}}{1 - \log \frac{x^2 - n}{t}} \Rightarrow \begin{cases} x^2 - n \geq 0 \text{ (1)} \\ x^2 - n > 0 \text{ (2)} \\ 1 - \log \frac{x^2 - n}{t} \neq 0 \text{ (3)} \end{cases}$

① $x^2 - n > 0, x^2 - n = 0 \Rightarrow \Delta = 1 \Rightarrow x_1, x_2 = \frac{1 \pm 1}{2} = \begin{cases} 0 \\ 1 \end{cases} \Rightarrow D_f = (-\infty, 0] \cup [1, +\infty)$ (1)

② $x^2 - n > 0, x^2 - n = 0 \Rightarrow \Delta = 9 + 0 = 9 \Rightarrow x_1, x_2 = \frac{3 \pm 3}{2} = \begin{cases} 0 \\ 3 \end{cases} \Rightarrow D_f = (-\infty, 0) \cup (3, +\infty)$ (2)

③ $1 - \log \frac{x^2 - n}{t} \neq 0 \Rightarrow \log \frac{x^2 - n}{t} \neq 1 \Rightarrow \frac{x^2 - n}{t} \neq 10 \Rightarrow x^2 - n - 10t \neq 0, \Delta = 4 + 160t \Rightarrow x_1, x_2 = \frac{2 \pm \sqrt{4 + 160t}}{2} = 1 \pm \sqrt{1 + 40t}$
 $\Rightarrow D_f = \mathbb{R} - \{-1, t\} \Rightarrow (1) \cap (2) \cap (3) \Rightarrow D_f = (-\infty, 0) \cup (3, +\infty) - \{t, -1\}$ ✓

⑩ الف) $y = \sqrt{r^{n-r} - 1} \Rightarrow r^{n-r} > 1 \Rightarrow r^{n-r} > r^p \Rightarrow r^{n-r} > r^p$
 $\Rightarrow n^p - r^{n-r} < 0 \Rightarrow (n-r)(n-1) < 0 \Rightarrow \frac{1}{+p} - \frac{1}{p+}$
 $\Rightarrow D_f = [1, r]$ ✓

②

ب) $\left(\frac{r_{n+\Delta}}{r_{n+k}} \right)! \Rightarrow \frac{r_{n+\Delta}}{r_{n+k}} \in W \Rightarrow r_{n+\Delta} \in r_{n+k}W + r_{n+k}W$

$\Rightarrow r_n - r_{n+k}W \in r_{n+k}W - \Delta \Rightarrow n(r - r_{n+k}W) \in r_{n+k}W - \Delta \Rightarrow n \in \frac{r_{n+k}W - \Delta}{r - r_{n+k}W}$

$\Rightarrow D_f = \left\{ n \mid n = \frac{r_{n+k}W - \Delta}{r - r_{n+k}W}, k \in W \right\}$ ✓