

$$\begin{aligned} & 2x^2 + y^2 - 4x + 8y + k = 0 \\ \Rightarrow & (\sqrt{2}x - \sqrt{2})^2 + (y + 4)^2 = 0 \\ \Rightarrow & 2x^2 + 2 - 4x + y^2 + 8y + 16 + k = 0 \\ \Rightarrow & k = 9 + 2 \Rightarrow \boxed{k = 11} \end{aligned}$$

$$\begin{aligned} f(x) &= \begin{cases} x^2 + 2x & ; x \geq a \\ 2x + a + 2 & ; x < a \end{cases} \xrightarrow{x=a} a^2 + 2a \geq 2a + a + 2 \\ \Rightarrow & a^2 + a - 2 \geq 0 \xrightarrow[\text{فرایب}]{\text{جمع}} a < -2 \text{ غلط} \\ \Rightarrow & f(1) = (2 \times 1) + 1 + 2 = 5 \end{aligned}$$

$$\begin{aligned} f(x) &= \begin{cases} 2x^2 - b & ; |x+1| \geq 2 \\ a + 2x & ; -2 \leq x \leq -1 \end{cases} \xrightarrow{x=-3} 2x^2 - b \geq a + 2x \\ \Rightarrow & 18 - b - a \geq -6 \Rightarrow \boxed{a + b \leq 24} \end{aligned}$$

$$\begin{aligned} y &= \sqrt{9x-a} + \sqrt{b-2x} \xrightarrow{x=2} \sqrt{12-a} + \sqrt{b-4} \\ \Rightarrow & a \leq 12, b \leq 6 \Rightarrow \frac{b}{a} \leq \frac{6}{12} = \boxed{\frac{1}{2}} \end{aligned}$$

$$\begin{aligned} \text{الف) } y &= \sqrt{\frac{x+1}{|x-3|}} + \sqrt{\frac{4-x}{x^2+2x+4}} \Rightarrow \left. \begin{aligned} x+1 &\geq 0 \Rightarrow x \geq -1 \\ x-3 &\leq 0 \Rightarrow x \leq 3 \\ 4-x &\geq 0 \Rightarrow x \leq 4 \\ x^2+2x+4 &\geq 0 \Rightarrow \text{دائما منفی همیشه مثبت} \end{aligned} \right\} \Rightarrow \text{اینها} \\ & \left. \begin{aligned} -1 &\leq x \leq 3 \\ x &\leq 4 \end{aligned} \right\} \Rightarrow [-1, 3] - \{3\} \end{aligned}$$

$$\text{ب) } y = \sqrt{[x]-4} + \sqrt{2-[x]} \Rightarrow \begin{aligned} & \begin{array}{|c|} \hline 0 \\ \hline \end{array} + \begin{array}{|c|} \hline 2 \\ \hline \end{array} \quad \vee \quad \begin{array}{|c|} \hline 1 \\ \hline \end{array} + \begin{array}{|c|} \hline 1 \\ \hline \end{array} \Rightarrow \text{تابع تکی} \\ & \text{اشتراک ندارد} \end{aligned}$$

الف) $\sqrt{x^2 - x - 6} \geq x^2 - x - 6 \Rightarrow (x-3)(x+2) \Rightarrow x \geq 3, -2$
 $\frac{x^2 - x - 6}{x^2 - 13x + 36} \Rightarrow x^2 \geq A \Rightarrow A^2 - 13A + 36 \Rightarrow (A-9)(A-4)$
 $\Rightarrow A \geq 9, 4 \Rightarrow x \geq \pm 2, \pm 3$
 $\Rightarrow x \in (-\infty, 2) \cup (2, 3) \cup (3, +\infty) + \{ -1, -1, +1, +1 \}$

ب) $\sqrt{x^2 - 2x^2 - 5x + 6} \rightarrow 0$ جذور $\Rightarrow x = 1, -2, 3$
 $\sqrt{x^2 + 2x^2 - 5x - 6} \rightarrow 0$ جذور $\Rightarrow x = -1, 2, -3$
 $\Rightarrow x \in \{1, -2, 3, -1, 2, -3\}$

الف) $y \geq \sqrt{-x} - 2 \Rightarrow x \leq -4 \Rightarrow (-\infty, -4]$

ب) $y \geq \frac{5x^2 + 6}{[x]^2 - 2[x] - 3} \Rightarrow x^2 - 2x - 3 \Rightarrow (x-3)(x+1) \Rightarrow x \geq 3, -1$
 $+ \begin{matrix} -1 & 0 & 3 & 4 \\ | & | & | & | \\ 0 & 1 & 0 & 1 \end{matrix} \Rightarrow D_F = \mathbb{R} - [-1, 0) \cup [3, 4]$

$y \geq \frac{\cot x + 1}{\tan x + 1} \geq \frac{\cos + 1}{\frac{\sin}{\cos} + 1} \Rightarrow \sin \neq 0, \cos \neq 0 \Rightarrow \mathbb{R} - \{ \frac{k\pi}{2}, k\pi - \frac{\pi}{2} \}$
 $\tan x + 1 \neq 0 \Rightarrow \tan x \neq -1$

$y \geq \sqrt{1 - \sin^2 x} \Rightarrow \sin^2 x \leq 1 \Rightarrow \sin^2 x \leq \frac{1}{4} \Rightarrow \sin x \leq \pm \frac{1}{2}$
 $\Rightarrow -\frac{1}{2} \leq \sin x \leq \frac{1}{2} \Rightarrow D_F = [k\pi - \frac{\pi}{6}, k\pi + \frac{\pi}{6}]$

الف) $y \geq \sqrt{1 - \log_{\frac{1}{2}}(x-1)} \Rightarrow x-1 > 0 \Rightarrow x > 1$
 $1 - \log_{\frac{1}{2}}(x-1) \geq 0 \Rightarrow \log_{\frac{1}{2}}(x-1) \leq 1$
 $\Rightarrow x-1 \geq \frac{1}{2} \Rightarrow x \geq \frac{3}{2}$
 $\Rightarrow D_F = [\frac{3}{2}, +\infty)$

ب) $\sqrt{x^2 - x} \Rightarrow x^2 - x \geq x(x-1) \Rightarrow x \geq 0, 1 \Rightarrow x \geq 0, 1$
 $\frac{1 - \log_{\frac{1}{2}}(x-1)}{1 - \log_{\frac{1}{2}}(x-1)} \Rightarrow x^2 - 2x \geq 0 \Rightarrow x \geq 0, 2$
 $\frac{1 - \log_{\frac{1}{2}}(x-1)}{1 - \log_{\frac{1}{2}}(x-1)} \neq 0 \Rightarrow \log_{\frac{1}{2}}(x-1) \neq 1 \Rightarrow x-1 \neq \frac{1}{2} \Rightarrow (x-1)(x+1) \neq 0$

الف) $y \geq \sqrt{x^2 - x^2} - 1 \Rightarrow x - x^2 \geq 3 \Rightarrow -x^2 + x - 3 \geq 0 \Rightarrow x \geq 1, 3$
 $\Rightarrow D_F = [1, 3]$

ب) $\left(\frac{x^2 + 5}{x^2 + 4} \right)! \Rightarrow \frac{x^2 + 5}{x^2 + 4} \in \mathbb{W}$
 $\Rightarrow D_F = \{ x \mid x = \frac{k^2 + 5}{k^2 + 4}, k \in \mathbb{W} \}$
 $\frac{x^2 + 5}{x^2 + 4} = \frac{k^2 + 5}{k^2 + 4} \Rightarrow x^2 - k^2 = 5 - 4 \Rightarrow x^2 - k^2 = 1 \Rightarrow x = \pm \sqrt{k^2 + 1}$

دائمه $x \in (-\infty, -3) \cup [-2, -1) \cup [1, 2) \cup [3, +\infty)$

الان سوال 6 بنویس :