محمدامین جعفری — تکلیف شماره ۲ — کلاس یازدهم بدر B

۱

$2x^2-4x+y^2+6y+k=0 \Rightarrow 2(x-1)^2+(y+3)^2=0$

$\Rightarrow 2x^2-4x+2+y^2+6y+9=0 \Rightarrow y=2+9=11$

۲

$x=a \Rightarrow x^2+4x=2x+a+2 \Rightarrow a^2+4a=2a+a+2 \Rightarrow a^2+a-2=0$

$\Rightarrow a=\begin{cases}1 \Rightarrow 1=1\\ -2 \Rightarrow 1>-2\end{cases} \Rightarrow f(1)=x^2+4x=(1)^2+4(1)=5$

۳

$|x+1|\geq 2 \begin{cases} x+1\geq 2 \Rightarrow x\geq 1\\ x+1\leq -2 \Rightarrow x\leq -3\end{cases}$

$x=-3 \Rightarrow 2x^2-b=a+2x \Rightarrow a+b=2x^2-2x=2(-3)^2-2(-3)=18+6$
$=24$

۴

$6x-a\geq 0 \Rightarrow x\geq \frac{a}{6}$ ① $\quad b-3x\geq 0 \Rightarrow x\leq \frac{b}{3}$ ②

$① \cap ② \longrightarrow \frac{a}{6}\leq x\leq \frac{b}{3} \Rightarrow \begin{cases}\frac{a}{6}=2 \Rightarrow a=12\\ \frac{b}{3}=2 \Rightarrow b=6\end{cases} \Rightarrow \frac{b}{a}=\frac{6}{12}=\frac{1}{2}$

۵

الف)

$\frac{x+1}{|x-3|}\geq 0$ $\quad$ (-1, 3: $-\ 0\ +\ |\ +$) $\quad x\in[-1,3)\cup(3,+\infty)$ ①

$\frac{6-x}{x^2+2x+4}\geq 0$ $\quad$ (6: $+\ 0\ -$) $\quad x\in(-\infty,6]$ ②

$① \cap ② \longrightarrow D=[-1,3)\cup(3,6]$

ب)

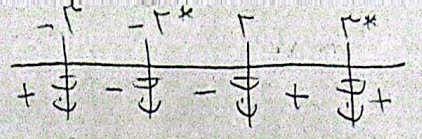
$[x]-4\geq 0 \Rightarrow [x]\geq 4 \Rightarrow x\geq 4$ ①

$2-[x]\geq 0 \Rightarrow 2\geq [x] \Rightarrow x<3$ ②

$① \cap ② \longrightarrow D=\emptyset$

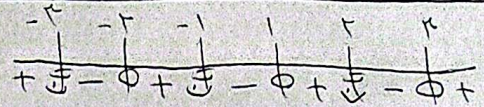
۶ الف)

$y=\sqrt{\frac{(x-2)(x+2)}{(x-3)(x+3)(x-1)(x+2)}}$



ب)

$y=\sqrt{\frac{(x-1)(x^2-x-6)}{(x+1)(x^2+x-2)}}=\sqrt{\frac{(x-1)(x-3)(x+2)}{(x+1)(x+2)(x-1)}}$



$\Rightarrow D=(-\infty,-3)\cup[-2,-1)\cup[1,2)\cup[3,+\infty)$

۷ الف)

$[-x]-1\geq 0 \Rightarrow [-x]\geq 1 \Rightarrow -x\geq 2 \Rightarrow x\leq -2$

ب)

$[x]^2-2[x]-3\neq 0 \Rightarrow ([x]+1)([x]-3)\neq 0 \Leftarrow \begin{matrix}[x]\neq -1 \Rightarrow x\in \mathbb{R}-[-1,0)\\ [x]\neq 3 \Rightarrow x\in \mathbb{R}-[3,4)\end{matrix}$

$\Rightarrow x\in(-\infty,-1)\cup[0,3)\cup[4,+\infty)$

۸ الف)

$\frac{\cos x+1}{\frac{\sin x}{\sin x}+1}$

$\sin x\neq 0 \Rightarrow x\neq k\pi$

$\frac{\sin x}{\cos x}+1\neq 0 \Rightarrow \frac{\sin x}{\cos x}\neq -1 \Rightarrow \sin x\neq -\cos x \Rightarrow x\neq 2k\pi-\frac{\pi}{4}$

$\cos x\neq 0 \Rightarrow x\neq 2k\pi+\frac{\pi}{2}$

$\Rightarrow x\in \mathbb{R}-\{2k\pi, 2k\pi-\frac{\pi}{4}, 2k\pi+\frac{\pi}{2}\}$

ب)

$1-4\sin^2 x\geq 0 \Rightarrow 4\sin^2 x\leq 1 \Rightarrow \sin^2 x\leq \frac{1}{4} \Rightarrow -\frac{1}{2}\leq \sin x\leq \frac{1}{2}$

$\Rightarrow x\in[2k\pi-\frac{\pi}{6}, 2k\pi+\frac{\pi}{6}]$

۹ الف)

$1-\log_{\frac{1}{2}}^{x-1}\geq 0 \Rightarrow \log_{\frac{1}{2}}^{x-1}\leq 1 \Rightarrow x-1\geq \frac{1}{2} \Rightarrow x\geq \frac{3}{2}$ ①

$x-1>0 \Rightarrow x>1$ ②

① ∩ ② → $x\geq \frac{3}{2}$

ب)

$x^2-x\geq 0 \Rightarrow x(x-1)\geq 0$ → $x\in(-\infty,0]\cup[1,+\infty)$ ①

$1-\log_{x^2-4x}\neq 0 \Rightarrow \log_{x^2-4x}\neq 1 \Rightarrow x^2-4x\neq 4 \Rightarrow x^2-4x-4\neq 0 \Rightarrow x\neq 2\pm 2\sqrt{2}$ ②

① ∩ ② → $D=(-\infty,-1)\cup(-1,0]\cup[1,2-2\sqrt{2})\cup(2+2\sqrt{2},+\infty)$

۱۰ الف)

$2^{4x-x^2}-8\geq 0 \Rightarrow 2^{4x-x^2}\geq 2^3 \Rightarrow 4x-x^2\geq 3 \Rightarrow x^2-4x+3\leq 0$

$\Rightarrow (x-1)(x-3)\leq 0$ → $x\in[1,3]$

ب)

$\frac{2n+5}{2n+2}\in \mathbb{N} \Rightarrow n\in -\frac{4\mathbb{N}-5}{2\mathbb{N}-2}\cdot\frac{5-4\mathbb{N}}{2\mathbb{N}-2} \Rightarrow \{n \mid n=\frac{5-4k}{2k-2}, k\in\mathbb{N}\}$