

$$2x^2 + y^2 + 4y - 8x + k = 0 \xrightarrow{\text{مربع کامل}} y^2 + 4y + x^2 - 2x + x^2 - 2x + k = 0$$

$$\Rightarrow y^2 + 4y + 9 + x^2 - 2x + 1 + x^2 - 2x + 1 = 0 \Rightarrow (y+3)^2 + 2(x-1)^2 = 0 \Rightarrow A \begin{cases} 1 \\ -3 \end{cases}$$

$$\Rightarrow y^2 + 4y + 2x^2 - 8x + 11 = 0 \Rightarrow k = 11 \quad \checkmark$$

$$f(x) = \begin{cases} x^2 + 8x; & x > a \\ 2x + a + 2; & x \leq a \end{cases} \quad \begin{matrix} \text{آرپیجی} \\ \text{آرپیجی} \\ \text{باید برابر} \\ \text{شوند} \end{matrix}$$

$$a^2 + 8a = 2a + a + 2 \Rightarrow a^2 + a - 2 = 0 \Rightarrow (a+2)(a-1) = 0 \Rightarrow \begin{cases} a = -2 \\ a = 1 \end{cases}$$

$$f(x) = \begin{cases} x^2 + 8x; & x > 1 \\ 2x + 3; & x \leq 1 \end{cases} \Rightarrow f(1) = 2(1) + 3 = 5 \quad \checkmark$$

$$f(x) = \begin{cases} 2x^2 - b; & |x+1| > 2 \\ a + 2x; & -3 \leq x \leq -1 \end{cases} \quad \begin{matrix} |x+1| > 2 \Rightarrow x+1 > 2 \Rightarrow x > 1 \\ x+1 < -2 \Rightarrow x < -3 \end{matrix}$$

$$\xrightarrow{\text{آرپیجی}} 2(-3)^2 - b = a + 2(-3) \Rightarrow 18 - b = a - 6 \Rightarrow a + b = 24 \quad \checkmark$$

دو عبارت
برابری شوند

$$y = \sqrt{4x-a} + \sqrt{b-3x} \Rightarrow 4x-a \geq 0 \Rightarrow 4x \geq a \Rightarrow x \geq \frac{a}{4}$$

$$\Rightarrow \frac{a}{4} = \frac{b}{3} \Rightarrow 3a = 4b \Rightarrow a = \frac{4}{3}b$$

$$\frac{b}{a} = \frac{b}{\frac{4}{3}b} = \frac{3}{4}$$

$$\frac{a}{4} = 2 \Rightarrow a = 12 \quad \frac{b}{3} = 2 \Rightarrow b = 6 \quad \frac{b}{a} = \frac{6}{12} = \frac{1}{2} \quad \checkmark$$

چون دامنه یک عضو است

$$a) y = \sqrt{\frac{x+1}{|x-3|}} + \sqrt{\frac{4-x}{2x^2+2x+8}} \Rightarrow \frac{x+1}{|x-3|} \geq 0 \Rightarrow \begin{matrix} -1 & 3 \\ | & | \\ - & + \end{matrix} \Rightarrow [1, +\infty) - \{3\}$$

$$\frac{4-x}{2x^2+2x+8} \geq 0 \Rightarrow \begin{matrix} 4 & - \\ | & | \\ + & - \end{matrix} \Rightarrow (-\infty, 4]$$

$$\Rightarrow [-1, 4] - \{3\} \quad \checkmark$$

$$b) y = \sqrt{[x] - 4} + \sqrt{2 - [x]} \Rightarrow [x] - 4 \geq 0 \Rightarrow [x] \geq 4 \Rightarrow x \geq 4$$

$$2 - [x] \geq 0 \Rightarrow 2 \geq [x] \Rightarrow 3 \geq x$$

$$D_f = (-\infty, 3) \cup [4, +\infty)$$

$$a) y = \sqrt{\frac{x^2 - x - 6}{x^2 - 13x^2 + 36}} \Rightarrow \frac{x^2 - x - 6}{x^2 - 13x^2 + 36} \geq 0 \Rightarrow \frac{(x-3)(x+2)}{(x^2-1)(x^2-6)} \geq 0 \Rightarrow \frac{(x-3)(x+2)}{(x-1)(x+3)(x-2)(x+2)} \geq 0$$

$$\Rightarrow \frac{-3}{+} \frac{-2}{-} \frac{+}{-} \frac{+}{+} \Rightarrow D_f = (-\infty, -3) \cup (2, +\infty) - \{2\} \checkmark$$

$$b) y = \sqrt{\frac{x^3 - 12x^2 - 6x + 6}{x^3 + 12x^2 - 6x - 6}} \Rightarrow \frac{x^3 - 12x^2 - 6x + 6}{x^3 + 12x^2 - 6x - 6} \geq 0 \Rightarrow \frac{(x-3)(x+2)(x-1)}{(x+3)(x-2)(x+1)} \geq 0$$

$$\frac{-3}{+} \frac{-2}{-} \frac{-1}{+} \frac{+}{-} \frac{+}{+} \Rightarrow D_f = (-\infty, -3) \cup (-2, -1) \cup (1, 2) \cup (3, +\infty) \checkmark$$

$$a) y = \sqrt{[-x] - 2} \Rightarrow [-x] - 2 \geq 0 \Rightarrow [-x] \geq 2 \Rightarrow [x] \leq -2 \Rightarrow x < -1 \quad x \leq -2$$

$$b) y = \frac{6x^2 + 6}{[x]^2 - 1[x] - 3} \Rightarrow [x]^2 - 1[x] - 3 \neq 0 \Rightarrow ([x] - 3)([x] + 1) \neq 0$$

$$[x] - 3 \neq 0 \Rightarrow [x] \neq 3 \Rightarrow 3 \leq x < 4 \Rightarrow \mathbb{R} - ([3, 4) \cup [-1, 0)) \checkmark$$

$$[x] + 1 \neq 0 \Rightarrow [x] \neq -1 \Rightarrow -1 \leq x < 0$$

$$a) y = \frac{\cot x + 1}{\tan x + 1} \Rightarrow \sin x \neq 0 \Rightarrow \cos x \neq 0 \Rightarrow \tan x + 1 \neq 0 \Rightarrow \tan x \neq -1$$

$$\Rightarrow D_f = \mathbb{R} - \left\{ \frac{k\pi}{2}, k\pi - \frac{\pi}{4} \right\} \checkmark$$

$$b) y = \sqrt{1 - \varepsilon \sin^2 x} \Rightarrow 1 - \varepsilon \sin^2 x \geq 0 \Rightarrow 1 \geq \varepsilon \sin^2 x \Rightarrow \frac{1}{\varepsilon} \geq \sin^2 x$$

$$\Rightarrow -\frac{1}{\sqrt{\varepsilon}} \leq \sin x \leq \frac{1}{\sqrt{\varepsilon}} \Rightarrow D_f = \left[k\pi - \frac{\pi}{2}, k\pi + \frac{\pi}{2} \right] \checkmark$$

$$a) y = \sqrt{1 - \log_{\frac{1}{2}} x - 1} \Rightarrow x - 1 \geq 0 \Rightarrow x \geq 1 \quad \frac{1}{2} \log_{\frac{1}{2}} x - 1 \geq 0 \Rightarrow 1 \geq \log_{\frac{1}{2}} x \Rightarrow \frac{1}{2} \leq x - 1$$

$$\Rightarrow \frac{3}{2} \leq x \Rightarrow D_f = \left[\frac{3}{2}, +\infty \right) \quad 0 < \sin x < 1 \quad \text{جواب}$$

$$b) y = \frac{\sqrt{x^2 - x}}{1 - \log_{\varepsilon} x^2 - 3x} \Rightarrow \frac{x^2 - x}{x^2 - 3x} \geq 0 \Rightarrow x(x-1) \geq 0 \Rightarrow \frac{+}{+} \frac{-}{-} \frac{+}{+} \Rightarrow D_f: (-\infty, 0) \cup (3, +\infty) - \{ \varepsilon, -1 \} \checkmark$$

$$a) y = \sqrt{\varepsilon x - x^2 - 1} \Rightarrow \varepsilon x - x^2 - 1 \geq 0 \Rightarrow \varepsilon x - x^2 \geq 1 \Rightarrow \varepsilon x - x^2 \geq \varepsilon^2$$

$$\varepsilon x - x^2 \geq \varepsilon^2 \Rightarrow 0 \geq x^2 - \varepsilon x + \varepsilon^2 \Rightarrow \frac{+}{+} \frac{-}{-} \frac{+}{+} \Rightarrow D_f = [1, 3] \checkmark$$

$$b) y = \left(\frac{3x + \omega}{x + \varepsilon} \right)! \Rightarrow \frac{3x + \omega}{x + \varepsilon} \in \mathbb{W} \Rightarrow 3x + \omega = xw + \varepsilon w \Rightarrow 3x - xw = \varepsilon w - \omega$$

$$\Rightarrow x(3 - w) = \varepsilon w - \omega \Rightarrow x = \frac{\varepsilon w - \omega}{3 - w} \Rightarrow D_f = \left\{ x \mid x = \frac{\varepsilon k - \omega}{3 - w}, k \in \mathbb{W} \right\} \checkmark$$