امیر مهدی سرزوئی یازدهم B تکلیف ۲۰

۱)

$$\frac{1}{x-1} = |x-1| \Rightarrow \frac{1}{x-1} = \pm\frac{(x-1)^2}{x-1}$$

$$\Rightarrow 0 = \frac{1 \pm (x-1)^2}{x-1} \Rightarrow \frac{(1+x-1)(2-x)}{x-1} = 0 \Rightarrow x = 2, 0$$

$$1 + (x-1)^2 > 0 \Rightarrow \times$$

$0 \rightarrow$ غ ق ق

$\boxed{\text{یک ریشه}}$

۲)

$$-2 < \frac{2x-1}{x} < 2 \Rightarrow -2 < 2 - \frac{1}{x} < 2$$

$$\Rightarrow -4 < -\frac{1}{x} < 0 \Rightarrow 4 > \frac{1}{x} > 0 \Rightarrow 4x > 1$$

$$\Rightarrow \boxed{x > \frac{1}{4}} \Rightarrow x \in \left(\frac{1}{4}, +\infty\right)$$

$$\frac{x-2}{2x+1} > 1 \quad , \quad \frac{x-2}{2x+1} < -1$$

$$\frac{x-2}{2x+1} - 1 > 0 \Rightarrow 0 < \frac{-x-3}{2x+1} \Rightarrow \begin{array}{ccccccc} -\infty & & -3 & & -\frac{1}{2} & & +\infty \\ & - & & + & & - & \end{array} \Rightarrow x \in \left(-3, -\frac{1}{2}\right)$$

$$\frac{x-2}{2x+1} + 1 < 0 \Rightarrow \frac{3x-1}{2x+1} < 0 \Rightarrow \begin{array}{ccccccc} -\infty & & -\frac{1}{2} & & \frac{1}{3} & & +\infty \\ & + & & - & & + & \end{array} \Rightarrow x \in \left(-\frac{1}{2}, \frac{1}{3}\right)$$

$$\left(-\frac{1}{2}, \frac{1}{3}\right) \cup \left(-3, -\frac{1}{2}\right) = \left(-3, \frac{1}{3}\right) - \left\{-\frac{1}{2}\right\} = D$$

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۳)

$$-\beta < x - \alpha < \beta \Rightarrow \alpha - \beta < x < \alpha + \beta$$

$$x^2 < x + 6 \quad \text{یا} \quad x + 6 < -x^2 \Rightarrow x^2 + x + 6 < 0 \xrightarrow{\Delta < 0} \varnothing$$

$$x^2 - x - 6 < 0 \Rightarrow \frac{+1 \pm 5}{2} = \begin{matrix} 3 \\ -2 \end{matrix} \qquad \begin{array}{ccccc} & -2 & & 3 & \\ + & & - & & + \end{array}$$

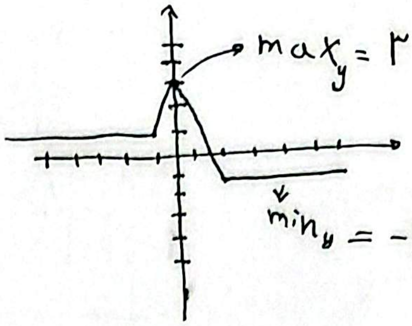
$$\Rightarrow (-2, 3) \cup \varnothing = D \Rightarrow \begin{matrix} \alpha - \beta = -2 \\ \alpha + \beta = 3 \end{matrix}$$

$$\boxed{\begin{matrix} \alpha = \frac{1}{2} \\ \beta = \frac{5}{2} \end{matrix}}$$

(۴)

$$y = \underset{-۱}{|x+۱|} - \underset{۰}{|۲x|} + \underset{۲}{|x-۲|}$$

$$\Rightarrow \begin{cases} x \geq ۲ \Rightarrow y = -۱ \\ ۰ \leq x \leq ۲ \Rightarrow y = -۲x + ۳ \\ -۱ \leq x \leq ۰ \Rightarrow y = ۲x + ۳ \\ x \leq -۱ \Rightarrow y = ۱ \end{cases}$$



$max_y + min_y = ۳ - ۱ = ۲$

(۵)

$f(x) = |\frac{۱}{۲}x| - ۲ \Rightarrow f(x+۴) = |\frac{۱}{۲}x+۲| - ۲$

$\Rightarrow f(x+۴)+۱ = |\frac{۱}{۲}x+۲| - ۱ \Rightarrow g(x) = |\frac{۱}{۲}x+۲| - ۱ = f(x+۴)+۱$

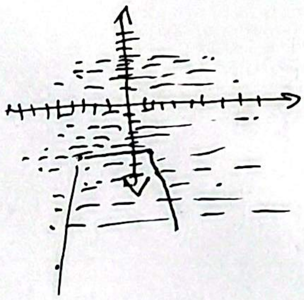
$g(x) = f(x) \Rightarrow |\frac{۱}{۲}x+۲| - ۱ = |\frac{۱}{۲}x| - ۲ \Rightarrow \underset{-۴}{|\frac{۱}{۲}x+۲|} - \underset{۰}{|\frac{۱}{۲}x|} + ۱ = ۰$

$\Rightarrow$ $-۱ = ۰$ $\quad -۴ \quad | \quad x + ۳ = ۰ \Rightarrow x = -۳ \checkmark \quad ۰ \quad | \quad ۳ = ۰ \times$ $\Rightarrow x = -۳ \Rightarrow y = -\frac{۱}{۲}$

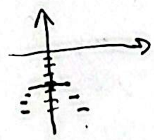
$\begin{vmatrix} -۳ \\ -\frac{۱}{۲} \end{vmatrix}$

(۶)

الف) $|۱-x| - |x+۴|$ $\quad$ $۲x+۳ \quad -۴ \quad | \quad -۵ \quad ۱ \quad | \quad -۳-۲x$



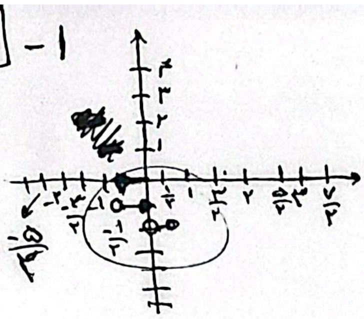
$\rightarrow$



$R = \{k \in \mathbb{Z} \mid k \leq -۵\}$

ب) $\frac{۱}{x} - ۱ - [\frac{۱}{x}] = \underbrace{t - [t] - ۱}_{R=[۰,۱)} \Rightarrow R = [-۱, ۰)$

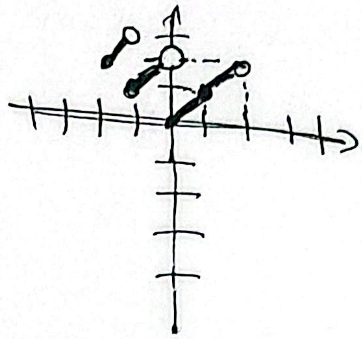
$[-2x]-1$



(۷)

$x-2\left[\frac{x}{2}\right]$

$-2 \;|\; x+4 \;|\; -1 \;|\; x+2 \;|\; 0 \;|\; x \;|\; 1 \;|\; x \;|\; 2$



خط

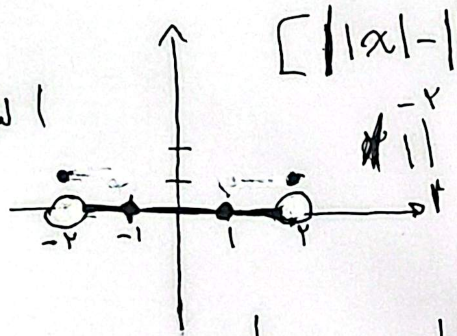
(۸)

$[\,||x|-1|\,]$

$-2 \quad -1 \quad 0 \quad +1 \quad +2$

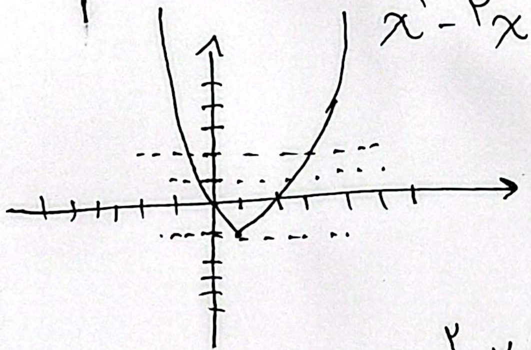
صفر

الف

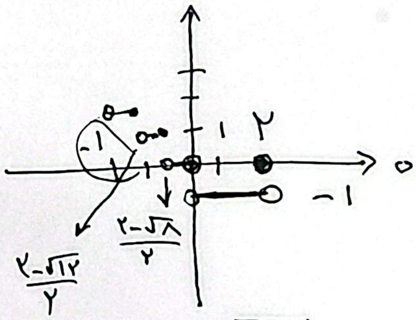


ب

x^2-2x



$\Rightarrow$



$f(-1) = x^2-2x = +3$

$x^2-2x=1 \Rightarrow x=\frac{2\pm\sqrt{8}}{2}$

$\Rightarrow x^2-2x=2 \Rightarrow \frac{2\pm\sqrt{12}}{2}$

(۹)

$b\in\mathbb{R},\ [a]\in\mathbb{Z} \Rightarrow 2.4 = b-[a]$

$\Rightarrow b-[b]=0.4$

$a\in\mathbb{R},\ [b]\in\mathbb{Z} \Rightarrow a-[a]=0.2$

$b-\cancel{[a]}+\cancel{[a]}-a=2.2$

$\Rightarrow b-a=2.2$

$a+[b]=4.2$

$\Rightarrow a+\cancel{[b]}+b-\cancel{[b]}=4.6$

$a+b=4.4$

$\Rightarrow \boxed{b=3.3 \Rightarrow a=1.1}$