

$$|a_1 - 1| \geq 0 \Rightarrow \frac{1}{a_1 - 1} > 0 \rightarrow a_1 > 1 \rightarrow |a_1 - 1| = a_1 - 1$$

$$\rightarrow \frac{1}{a_1 - 1} = a_1 - 1 \rightarrow a_1^2 - 2a_1 + 1 = 1 \rightarrow a_1^2 - 2a_1 = 0 \Rightarrow a_1(a_1 - 2) = 0$$

$$\rightarrow a_1 < \begin{matrix} 0 \times (a_1 > 1) \\ 2 \checkmark \end{matrix} \Rightarrow a_1 = \{2\} \quad \text{یک جواب دارد.}$$

الف) $-2 < \frac{2a_1 - 1}{a_1} < 2 \rightarrow \frac{2a_1 - 1}{a_1} < 2 \rightarrow \frac{2a_1 - 1}{a_1} - 2 < 0 \rightarrow \frac{-1}{a_1} < 0 \rightarrow a_1 > 0$ (I)
 $-2 < \frac{2a_1 - 1}{a_1} \rightarrow 0 < \frac{2a_1 - 1}{a_1} + 2 \rightarrow 0 < \frac{2a_1 - 1}{a_1} + \frac{2}{1} \rightarrow (-\infty, 0) \cup (\frac{1}{2}, +\infty)$
 In II $\Rightarrow a_1 = (\frac{1}{2}, +\infty)$

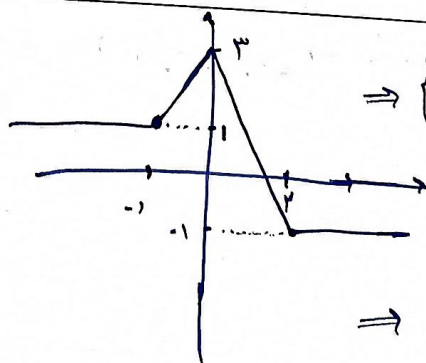
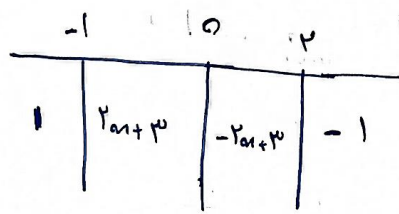
ب) $\frac{a_1 - 2}{2a_1 + 1} > 1 \rightarrow \frac{a_1 - 2 - 2a_1 - 1}{2a_1 + 1} > 0 \rightarrow \frac{-a_1 - 3}{2a_1 + 1} > 0 \rightarrow \frac{-a_1 - 3}{-2a_1 - 1} \Rightarrow (-3, -\frac{1}{2})$

$\frac{a_1 - 2}{2a_1 + 1} < -1 \rightarrow \frac{a_1 - 2 - 2a_1 - 1}{2a_1 + 1} < 0 \rightarrow \frac{-a_1 - 3}{2a_1 + 1} < 0 \rightarrow \frac{-a_1 - 3}{-2a_1 - 1} \Rightarrow (-\frac{1}{2}, \frac{1}{2}) \cup (-3, -\frac{1}{2}) \cup (-\frac{1}{2}, \frac{1}{2})$

$a_1^2 < a_1 + 4 \rightarrow a_1^2 - a_1 - 4 < 0 \rightarrow \frac{-1 \pm \sqrt{1 + 16}}{2} \rightarrow (-3, 4)$

$a_1 + 4 < -a_1^2 \rightarrow a_1^2 + a_1 + 4 < 0 \rightarrow \Delta < 0, a > 0 \Rightarrow \text{موجب ندارد.}$

$\Rightarrow -3 < a_1 < 2 \rightarrow |a_1 - \alpha| < \beta \rightarrow -\beta < a_1 - \alpha < \beta \rightarrow -\beta + \alpha < a_1 < \beta + \alpha$
 $\Rightarrow \begin{cases} \beta + \alpha = 2 \\ -\beta + \alpha = -3 \end{cases} \xrightarrow{+} 2\alpha = -1 \rightarrow \alpha = -\frac{1}{2}$
 $\beta = -\frac{5}{2} \quad |a_1 + \frac{1}{2}| < -\frac{\Delta}{2} \Rightarrow \alpha = -\frac{1}{2}$



$$\Rightarrow \begin{cases} \max = 3 \\ \min = -1 \end{cases}$$

$$\Rightarrow 3 + (-1) = 2$$

$$y = \left| \frac{a_1}{2} \right| - 2$$

$$y' = \left| \frac{a_1 + 2}{2} \right| - 1 \Rightarrow \left| \frac{a_1}{2} \right| - 2 = \left| \frac{a_1 + 2}{2} \right| - 1 \rightarrow \left| \frac{a_1}{2} \right| = \left| \frac{a_1 + 2}{2} \right| + 1$$

$a_1 > 0 \rightarrow \frac{a_1}{2} = \frac{a_1 + 2}{2} + 1 \rightarrow 0 \neq 3 \times$
 $-2 < a_1 < 0 \rightarrow \frac{-a_1}{2} = \frac{a_1 + 2}{2} + 1 \rightarrow a_1 = -3 \checkmark$
 $a_1 < -2 \rightarrow \frac{-a_1}{2} = \frac{-a_1 - 2}{2} - 1 \rightarrow 0 \neq -3 \times$

$$f(x), |x| - |x+2|$$

الف)

$$\frac{-x}{\omega} \quad |x| - |x+2|$$



$$R' = [-\omega, \omega]$$

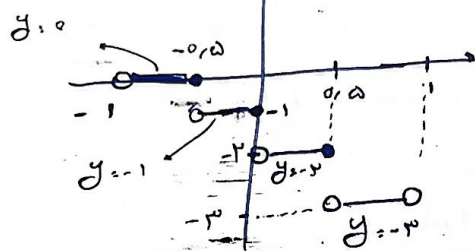
$$R = \{-\omega, -\varepsilon, -3, -1, -1, 0, 1, 3, \varepsilon, \omega\}$$

$$\rightarrow \left\lfloor \frac{a+1}{n} \right\rfloor = \left\lfloor 1 + \frac{1}{n} \right\rfloor = 1 + \left\lfloor \frac{1}{n} \right\rfloor$$

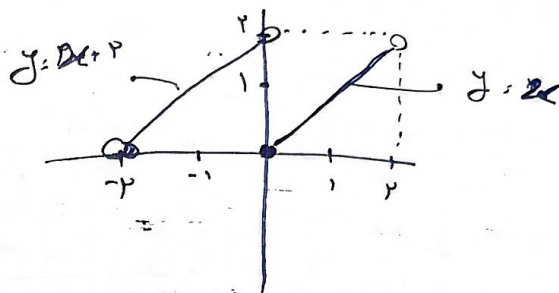
$$\rightarrow f(n) = \frac{1}{n} - \left\lfloor \frac{1}{n} \right\rfloor - 1 \rightarrow R = [-1, 0)$$

$$R = [0, 1)$$

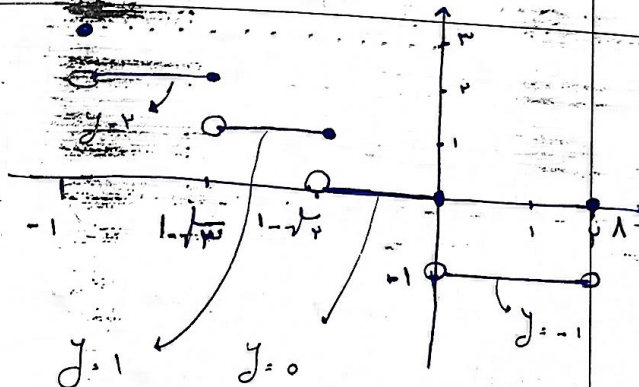
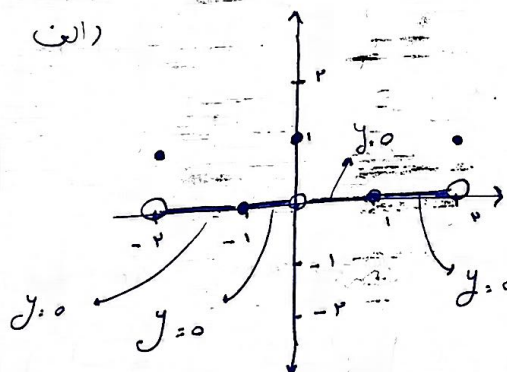
الف)



$$y = r \left(\frac{a}{r} - \left\lfloor \frac{a}{r} \right\rfloor \right)$$



الف)



$$\begin{aligned} a + [b] : \varepsilon, r &\rightarrow a : \dots, r \Rightarrow [a] + [b] : r \\ b - [a] : r, \varepsilon &\rightarrow b : \dots, \varepsilon \Rightarrow [b] - [a] : r \end{aligned} \quad \left. \begin{aligned} [b] = r &\Rightarrow b : r, \varepsilon \\ [a] = 1 &\Rightarrow a : 1, r \end{aligned} \right\}$$

$$\rightarrow a + b : \boxed{r, r}$$

$$1 < \sqrt{r} < 2 \rightarrow -1 < -\sqrt{r} < -2 \rightarrow 0 < 1 - \sqrt{r} < -1 \rightarrow 0 < (1 - \sqrt{r})^4 < 1$$

$$(1 + \sqrt{r})^9 = 198 - \underbrace{(1 - \sqrt{r})^9}_{(0, 1)} \Rightarrow 198 < (1 + \sqrt{r})^9 < 199$$

$$\rightarrow \left\lfloor (1 + \sqrt{r})^9 \right\rfloor = \boxed{198}$$