

$\frac{1}{x-1} = |x-1|$
 $x < 0 \Rightarrow \frac{1}{x-1} = -(x-1) \Rightarrow \frac{1}{x-1} = -x+1 \Rightarrow \frac{1}{x-1} + x - 1 = 0$
 $\Rightarrow \frac{1+x^2-x-1}{x-1} = 0 \Rightarrow \frac{x^2-x}{x-1} = 0 \Rightarrow \frac{x(x-1)}{x-1} = 0 \Rightarrow x=0$
 if $x > 0 \Rightarrow \frac{1}{x-1} = x-1 \Rightarrow (x-1)^2 = 1 \Rightarrow x-1 = \pm 1$
 $\Rightarrow x=2$ or $x=0$

الف) $|\frac{2x-1}{x}| < 2$
 $-2 < \frac{2x-1}{x} < 2$
 $\frac{2x-1}{x} > -2 \Rightarrow \frac{2x-1+2x}{x} > 0 \Rightarrow \frac{4x-1}{x} > 0$
 $\frac{2x-1}{x} < 2 \Rightarrow \frac{2x-1-2x}{x} < 0 \Rightarrow \frac{-1}{x} < 0 \Rightarrow x > 0$
 $\Rightarrow \frac{4x-1}{x} > 0 \Rightarrow x > \frac{1}{4}$
 ب) $|\frac{x-2}{2x+1}| > 1$
 $\frac{x-2}{2x+1} > 1 \Rightarrow \frac{x-2-2x-1}{2x+1} > 0 \Rightarrow \frac{-x-3}{2x+1} > 0$
 $\frac{x-2}{2x+1} < -1 \Rightarrow \frac{x-2+2x+1}{2x+1} < 0 \Rightarrow \frac{3x-1}{2x+1} < 0$
 $\Rightarrow x < \frac{1}{3}$
 $\Rightarrow (-3, -\frac{1}{2}) \cup (-\frac{1}{2}, \frac{1}{3})$

$x^2 < |x+6|$
 if $x < -6 \Rightarrow x^2 < -x-6 \Rightarrow x^2+x+6 < 0 \Rightarrow \Delta < 0$
 if $x > -6 \Rightarrow x^2 < x+6 \Rightarrow x^2-x-6 < 0 \Rightarrow (x-3)(x+2) < 0 \Rightarrow x \in (-2, 3)$
 $\Rightarrow x \in (-2, 3)$

$y = |x+1| - |2x| + |x-2|$
 $-1 \leq x \leq 0 \Rightarrow -2 \leq 2x \leq 0 \Rightarrow 1 \leq 2x+3 \leq 3$
 $0 \leq x \leq 2 \Rightarrow -4 \leq -2x \leq 0 \Rightarrow -1 \leq 3-2x \leq 3$
 $\Rightarrow y_{max} = 3$
 $y_{min} = -1$

نمودار اولیه: $y = |\frac{1}{x}| - 2$
 نمودار جدید: $y = |\frac{1}{x}(x+4)| - 2 + 1 = |\frac{1}{x}(x+2)| - 1$
 $|\frac{1}{x}| - 2 = |\frac{1}{x}(x+2)| - 1$
 $\Rightarrow x = -3 \Rightarrow \frac{1}{-3} - 2 = \frac{1}{-3} - 1$
 $\Rightarrow -\frac{1}{3} = -\frac{1}{3}$

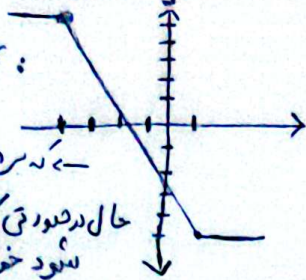
$$الف) y = [1-x] - [x+4]$$

ممنوعه: $|1-x| - |x+4|$

که در آن $[-1, 0]$

حال در صورتی که برآنت در نظر گرفته شود خواهیم داشت:

$$R_f = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}$$



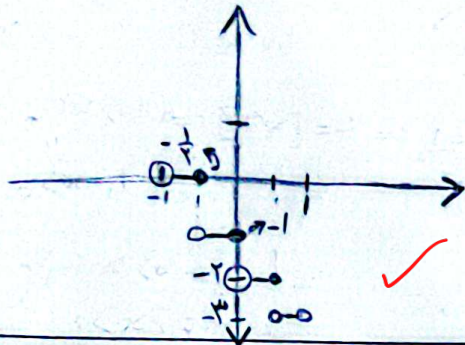
$$ب) f(x) = \frac{1}{x} - \left[\frac{x+1}{x} \right]$$

$$f(x) = \frac{1}{x} - \left[\frac{1}{x} \right] - 1 \quad (x \neq 0)$$

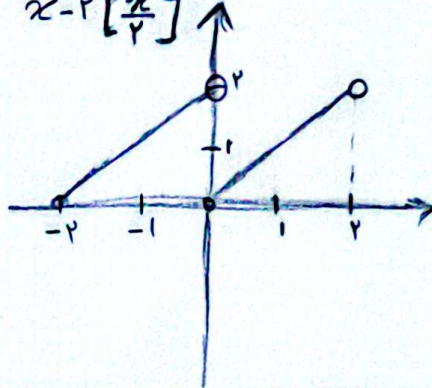
$$0 \leq \frac{1}{x} - \left[\frac{1}{x} \right] < 1 \Rightarrow -1 \leq \frac{1}{x} - \left[\frac{1}{x} \right] - 1 < 0$$

$$\Rightarrow R_f = [-1, 0)$$

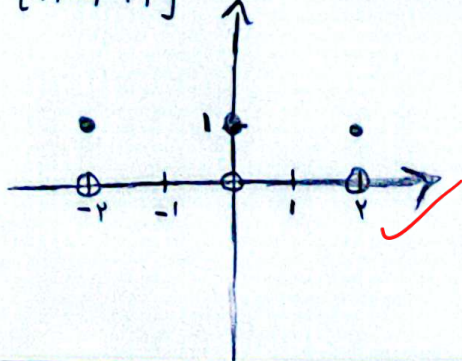
$$y = [-2x] - 1$$



$$ب) y = x - 2 \left[\frac{x}{2} \right]$$

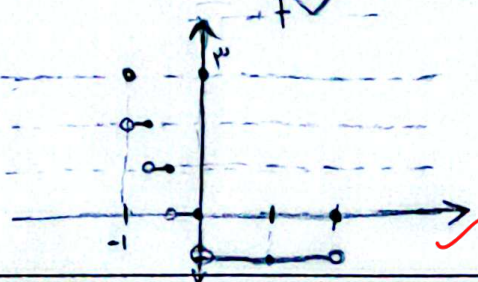


$$f = [|x| - 1]$$



$$y = [x^2 - 2x]$$

$$x^2 - 2x:$$



$$a + [b] = 4, 2$$

$$b - [a] = 2, 4$$

$$[a + [b]] = [4, 2] \Rightarrow [a] + [b] = 4$$

$$[b - [a]] = [2, 4] \Rightarrow [b] - [a] = 2 \Rightarrow [b] = 3, [a] = 1$$

$$\Rightarrow \begin{cases} a + 3 = 4, 2 \Rightarrow a = 1, 2 \\ b - 1 = 2, 4 \Rightarrow b = 3, 4 \end{cases} \Rightarrow a + b = 1, 2 + 3, 4 = [4, 4]$$

$$1 + \sqrt{2} = a, 1 - \sqrt{2} = b$$

$$a^4 + b^4 = 191$$

$$a^4 = (1 + \sqrt{2})^4$$

$$a + b = 2, ab = (1 + \sqrt{2})(1 - \sqrt{2}) = 1 - 2 = -1$$

$$b = 1 - \sqrt{2} \rightarrow |b| < 1 \Rightarrow b^4 = \text{عدد کوچکتر از یک است}$$

$$a^4 = 191 - b^4 \Rightarrow 191 < a^4 < 191 \Rightarrow [a^4] = 191$$

$$\text{پس: } 1 - \sqrt{2} = 1 - \sqrt{2} \times \frac{1 + \sqrt{2}}{1 + \sqrt{2}} = \frac{-1}{1 + \sqrt{2}} \Rightarrow (1 + \sqrt{2})^4 + \left(\frac{-1}{1 + \sqrt{2}} \right)^4 = 191$$

$$(1 + \sqrt{2})^4 = A \Rightarrow A + \frac{1}{A} = 191 \xrightarrow{\times A} A^2 - 191A + 1 = 0$$

$$A = \frac{191 \pm \sqrt{191^2 - 4}}{2} = 99 \pm 10\sqrt{2}$$

$$99 + 10\sqrt{2} < 99 - 10\sqrt{2}$$

$$[(1 + \sqrt{2})^4] = [99 + 10\sqrt{2}] = 99 + 91 = 190$$