

$$\frac{1}{x-1} \geq |x-1| \xrightarrow{\text{if } x > 1} \frac{1}{x-1} \leq x-1 \rightarrow x^2 - 2x + 1 \leq x-1 \rightarrow x^2 - 3x + 2 \leq 0$$

نقش ۰ $x=0$ $x=2$

$$\xrightarrow{\text{if } x < 1} \frac{1}{x-1} \geq -x+1 \rightarrow |2-x+1|$$

$$x^2 - 2x + 2 > 0 \rightarrow \text{ریشه ندارد} \rightarrow \text{همیشه مثبت و از ۰ بزرگتر}$$

$$\frac{|2x-1|}{x} \geq 2 \rightarrow -2 \leq \frac{2x-1}{x} \leq 2$$

$$\rightarrow -2 \leq 2 - \frac{1}{x} \leq 2$$

$$-2 \leq 2 - \frac{1}{x}$$

$$-4 \leq -\frac{1}{x} \rightarrow \frac{1}{x} > 4 \rightarrow \frac{1}{x} > 4 \rightarrow x < \frac{1}{4}$$

$$2 - \frac{1}{x} \leq 2 \rightarrow -\frac{1}{x} \leq 0$$

$$\frac{|x-2|}{2x+1} \geq 1 \rightarrow \frac{x-2}{2x+1} \geq 1 \quad \frac{x-2}{2x+1} \leq -1$$

$$x > -\frac{1}{2}, \quad x < \frac{1}{2}$$

$$\frac{x-2}{2x+1} \geq 1 \rightarrow x-2 \geq 2x+1 \rightarrow x \leq -3$$

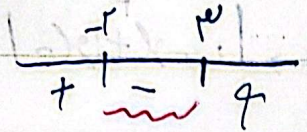
$$\frac{x-2}{2x+1} \leq -1 \rightarrow x-2 \leq -2x-1 \rightarrow 3x \leq 1 \rightarrow x \leq \frac{1}{3}$$

$$\frac{2x+1}{x} \geq \frac{1}{x} \rightarrow 2x+1 \geq 1 \rightarrow 2x \geq 0 \rightarrow x \geq 0$$

$$\frac{-1}{2x+1} \geq \frac{1}{x} \rightarrow x > -\frac{1}{2}$$

$$x^2 \leq |x+4| \rightarrow x^2 \leq x+4 \rightarrow -4$$

$$x^2 - x - 4 \leq 0 \rightarrow (x-2)(x+2) \leq 0$$



$$-2 \leq x \leq 2 \quad I$$

$$x^2 - x - 4 \rightarrow x^2 + x + 4 > 0 \quad \text{The discriminant is negative, so the expression is always positive.}$$

$$\text{The discriminant is negative, so the expression is always positive.} \quad II$$

$$I \text{ and } II \rightarrow x^2 \leq x+4 \rightarrow$$

$$|x-2| \leq \beta \rightarrow -\beta+2 \leq x \leq \beta+2$$

$$\begin{cases} \beta+2 = 2 \\ -\beta+2 = -2 \end{cases} \rightarrow$$

$$\boxed{x^2 \leq 1} \rightarrow x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

$$f_2 |x+1| - |2x| + |x-2| \rightarrow -16$$

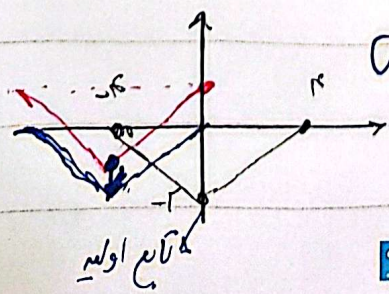
$$\text{if } x \geq 2 \rightarrow x+1 - 2x - x+2 = -2 = 0$$

$$\text{if } 1 \leq x < 2 \rightarrow x+1 - 2x - x+2 = -2x+3 = 0 \rightarrow x = \frac{3}{2}$$

$$\text{if } 0 \leq x < 1 \rightarrow x+1 + 2x - x+2 = 2x+3 = 0 \rightarrow x = -\frac{3}{2}$$

$$-1 > x \rightarrow -x-1 + 2x - x+2 = 1 = 0$$

$$\max y = 2, \min y = -1 \rightarrow \max y + \min y = 2$$



$$f_2 \left| \frac{1}{2}x+2 \right| - |x| = \text{تابع اوله}$$

$$\left| \frac{1}{2}x+2 \right| - |x| = \left| \frac{1}{2}x \right| - 2$$

$$\left| \frac{1}{2}x+2 \right| > \left| \frac{1}{2}x \right| - 2$$

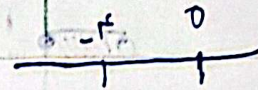
Subo
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اداره درصفت بعدی

$$\left| \frac{1}{x+2} \right| - \left| \frac{1}{x} \right| \geq -1$$

$$\text{if } x \geq 0 \rightarrow \frac{1}{x+2} - \frac{1}{x} \geq -1$$

اثبات آوردن این قضیه ضروری نمی‌کنیم



$$\frac{1}{x+2} - \frac{1}{x} \geq -1 \quad \times \text{SOS}$$

$$\text{if } x < -2 \rightarrow \frac{1}{x+2} + \frac{1}{x} \geq -1 \rightarrow \frac{x+2}{x(x+2)} + \frac{1}{x} \geq -1 \rightarrow \frac{x+2}{x} + 1 \geq -1$$

if $x < -2$

$$\text{if } -2 < x < 0 \rightarrow \frac{1}{x+2} - \frac{1}{x} \geq -1 \rightarrow \frac{x+2}{x(x+2)} - \frac{1}{x} \geq -1 \rightarrow \frac{x+2}{x} - 1 \geq -1$$

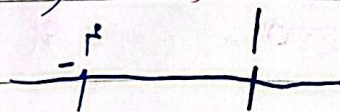
$$\left| \frac{1}{x-3} + 2 \right| - \left| \frac{1}{x} \right| \geq -1 \quad \text{و} \quad \left| \frac{1}{x-3} - 2 \right| - \left| \frac{1}{x} \right| \geq -1$$

بنابراین دو نمودار باطل می‌شود - متقاطع هستند

$$\left(-\frac{1}{2}, -2 \right)$$

$$f(x) = \left[\left| \frac{1}{x} \right| - \left| \frac{1}{x+2} \right| \right] \quad \text{انف}$$

محصول ضروری نمی‌کنیم



$$\text{I if } x > 0 \rightarrow \frac{1}{x} - \frac{1}{x+2} \geq -1 \rightarrow \frac{x+2}{x(x+2)} - \frac{1}{x} \geq -1 \rightarrow \frac{x+2}{x} - 1 \geq -1$$

$$\text{II if } x < -2 \rightarrow \frac{1}{x} + \frac{1}{x+2} \geq -1 \rightarrow \frac{x+2}{x(x+2)} + \frac{1}{x} \geq -1 \rightarrow \frac{x+2}{x} + 1 \geq -1$$

$$\text{III if } -2 < x < 0 \rightarrow \frac{1}{x} - \frac{1}{x+2} \geq -1 \rightarrow \frac{x+2}{x(x+2)} - \frac{1}{x} \geq -1 \rightarrow \frac{x+2}{x} - 1 \geq -1$$

حال برای حل II، ابتدا آزمون داخل [] قرار دهیم

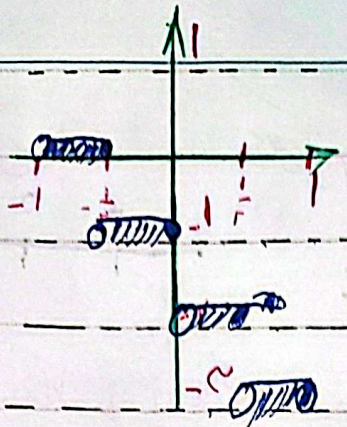
$$\frac{1}{x} + \frac{1}{x+2} \geq -1 \rightarrow \frac{x+2}{x(x+2)} + \frac{1}{x} \geq -1 \rightarrow \frac{x+2}{x} + 1 \geq -1$$

$$R_f = \{ -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5 \}$$

$$\text{ب) } f(x) = \frac{1}{x} - \left[\frac{x+1}{x} \right] \rightarrow \frac{1}{x} - \left[1 + \frac{1}{x} \right] = \frac{1}{x} - \left[\frac{x+1}{x} \right] - 1 \rightarrow$$

$$\left| \frac{1}{x} - \left[\frac{x+1}{x} \right] \right| \geq -1 \rightarrow \left| \frac{1}{x} - \left[\frac{x+1}{x} \right] \right| \geq -1 \quad R_f = [-1, 0)$$

الف)



$$y = \begin{cases} -1 & x \in (-1, 0) \end{cases}$$

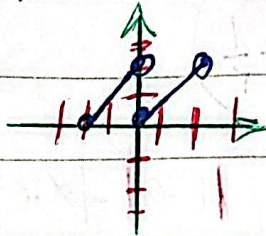
$$-1 < x < -\frac{1}{2} \Rightarrow 0$$

$$\begin{cases} -\frac{1}{2} < x < 0 \Rightarrow -1 \\ 0 < x < \frac{1}{2} \Rightarrow -2 \\ \frac{1}{2} < x < 1 \Rightarrow -0 \end{cases}$$

ب) $y = \begin{cases} x \\ x-2 \end{cases} \quad x \in (-1, 1)$

$$-2 < x < -1 \Rightarrow -1 < x < 0 \Rightarrow -1 < x < 1 \Rightarrow 0$$

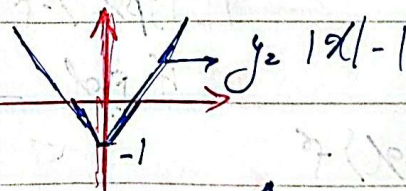
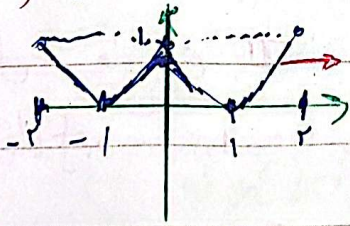
$$f(x) = \begin{cases} x \\ x-2 \end{cases}$$



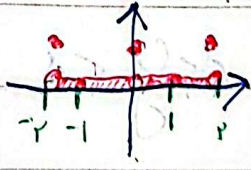
if $x < -1$ then $f(x) = x-2$

if $x > 1$ then $f(x) = x$

الف) $y = |x| - 1$



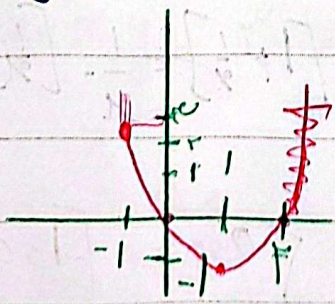
-1



ب) $y = \begin{cases} x \\ x-2 \end{cases} \quad x \in [-1, 1]$

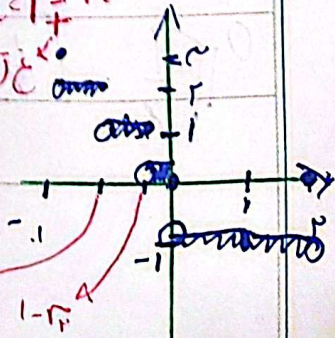
ابتدا می بینیم که این تابع یک تابع زوج است

$$\frac{f(-x)}{f(x)} = \frac{f(x)}{f(x)} = 1$$



$$y = x^2 - 2x \Rightarrow x(x-2) = 0 \Rightarrow x = 0 \text{ or } x = 2$$

$$y = x^2 - 2x \Rightarrow x(x-2) = 0 \Rightarrow x = 0 \text{ or } x = 2$$



$$a + [b] \in \mathbb{Z} \rightarrow \frac{1}{10} \in \mathbb{Z}$$

$$b - [a] \in \mathbb{Z} \rightarrow \frac{4}{10} \in \mathbb{Z}$$

با توجه به اینکه جمع جزایهای آن دو کسرها است

$$a \in \mathbb{Z} + \frac{1}{10}, b \in \mathbb{Z} + \frac{4}{10}$$

$$[a+b] = [a] + [b]$$

$$[a] + [b] \in \mathbb{Z} \quad (19)$$

$$\left[\underbrace{\quad}_{\substack{\uparrow \\ \text{اجزاء صحیح } a}} + \frac{1}{10} + \underbrace{\quad}_{\substack{\uparrow \\ \text{اجزاء صحیح } b}} + \frac{4}{10} \right] \in \mathbb{Z} \rightarrow$$

$$\underbrace{\quad}_{\substack{\uparrow \\ \text{اجزاء صحیح } a}} + \underbrace{\quad}_{\substack{\uparrow \\ \text{اجزاء صحیح } b}} \in \mathbb{Z}$$

$$[a] + [b] \in \mathbb{Z} \Rightarrow$$

$$a + b \in \mathbb{Z}$$

اجزاء صحیح a اجزاء صحیح b

$$[(1 + \sqrt{2})^4] \in \mathbb{Z} \quad (19)$$

$$[(1 + \sqrt{2})^4] + [(1 - \sqrt{2})^4] + \underbrace{(a)}_{\substack{\uparrow \\ \text{جزء صحیح } a}} + \underbrace{(b)}_{\substack{\uparrow \\ \text{جزء صحیح } b}} \in \mathbb{Z}$$