نام و نام خانوادگی .... پاسخنامه تشریحی تکلیف شماره .... کلاس ....

۱۹,۲۵

۱ (۲)

$\frac{1}{x-1} = |x-1|$

$x>1 \rightarrow \frac{1}{x-1} = x-1 \rightarrow 1 = x^2-2x+1 \rightarrow x^2-2x=0 \begin{cases} x=0 \; ✗ \\ x=2 \; ✓ \end{cases}$ در اعداد مثبت

$x<1 \rightarrow \frac{1}{x-1} = 1-x \rightarrow 1 = -x^2+2x-1 \rightarrow x^2-2x+2=0$

$\Delta = b^2-4ac \rightarrow (-2)^2-4(1)(2) = 4-8 = -4 \; ✗$

یک ریشه دارد ✓

۲ (۱,۲۵)

الف) $-2 < \frac{2x-1}{x} < 2 \rightarrow -2 < 2-\frac{1}{x} < 2 \rightarrow -4 < -\frac{1}{x} < 0 \rightarrow 4 > \frac{1}{x} > 0$

$\frac{1}{4} < x < +\infty \rightarrow \frac{1}{4} < x$ ✓

$\frac{x-2}{2x+1} < -1 \rightarrow \frac{1}{2} - \frac{2.5}{2x+1} > -1 \rightarrow \frac{2.5}{2x+1} > \frac{3}{2} \rightarrow 2x+1 < \frac{15}{4} \rightarrow \cancel{x < \frac{11}{8}}$

ب) $\frac{x-2}{2x+1} > 1 \rightarrow \frac{x+1.5-2.5}{2x+1} > 1 \rightarrow \frac{1}{2} - \frac{2.5}{2x+1} > 1 \rightarrow \frac{-2.5}{2x+1} > \frac{1}{2} \rightarrow \frac{2.5}{2x+1} < -\frac{1}{2}$

$\rightarrow \frac{2x+1}{2.5} > -2 \rightarrow 2x+1 > -5 \rightarrow 2x > -6 \rightarrow x > -3$

~~۵~~ ۴ (۲)

$|x+1| - |2x| + |x-2| \begin{cases} x+1-2x+x-2 = -1 & x \geq 2 \\ x+1-2x-x+2 = -2x+3 & 2 > x > 0 \rightarrow (-1,3) \\ x+1+2x-x+2 = 2x+3 & 0 \geq x > -1 \rightarrow (1,3] \\ -x-1+2x-x+2 = 1 & x \leq -1 \end{cases}$

$R_f = R_1 + R_2 + R_3 + R_4 = \{-1\} \cup (-1,3) \cup (1,3] \cup \{1\} = [-1,3]$

$3 + -1 = 2$ ✓

~~۴~~ ۳ (۲)

$x^2 < x+6 \qquad x^2-x-6 < 0 \qquad \Delta = 1-4(1)(-6) = 25$

$\frac{1 \pm \sqrt{25}}{2} \begin{cases} 3 \\ -2 \end{cases} \qquad + \; | \; - \; | \; + \rightarrow x \in (-2,3)$

$x^2 > -x-6 \rightarrow x^2+x+6 > 0 \qquad \Delta = b^2-4ac = 1-4(1)(6) = -23 \; ✗$

$\frac{3+-2}{2} = \frac{1}{2} \qquad \frac{3--2}{2} = 2\frac{1}{2} \qquad |x-\frac{1}{2}| < 2\frac{1}{2} \rightarrow a = \frac{1}{2}$ ✓

۵ (۲)

تابع جدید $= |\frac{1}{2}x+2|-1 \qquad |\frac{1}{2}x+2|-1 = |\frac{1}{2}x|-2 \rightarrow |\frac{1}{2}x+2| = |\frac{1}{2}x|-1$

$|\frac{1}{2}x+2| - |\frac{1}{2}x| = -1 \begin{cases} \frac{1}{2}x+2-\frac{1}{2}x = -1 \rightarrow 2 = -1 \; ✗ & x \geq 0 \\ \frac{1}{2}x+2+\frac{1}{2}x = -1 \rightarrow x+2 = -1 \rightarrow x = -3 \; ✓ & 0 > x > -4 \\ -\frac{1}{2}x-2+\frac{1}{2}x = -1 \rightarrow -2 = -1 \; ✗ & x \leq -4 \end{cases}$

$-3 \rightarrow |-\frac{3}{2}+\frac{4}{2}| - |-\frac{3}{2}| = -1$

$\frac{1}{2} - \frac{3}{2} = -1$ ✓

۶

الف) $|4-x|-|x+4|$
$$\begin{cases} -1+x-x-4=-5 & x\ge 1 \\ 1-x-x-4=-2x-3 & 1>x\ge -4 \\ 1-x+x+4=5 & x\le -4 \end{cases}$$

$[R_f]=[R_1+R_2+R_3]$

$[\{-5\}\cup(-5,5)\cup\{5\}]=[[-5,5]]=\{-5,-4,-3,-2,-1,0,1,2,3,4,5\}$

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ب) $\frac{1}{x}-[\frac{x}{x}+\frac{1}{x}]=\frac{1}{x}-[\frac{1}{x}]-1=[0,1)-1=[-1,0)$

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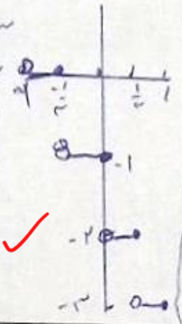
الف) $-1>x>1 \to -2<-2x<2$

$-2<-2x<-1 \to [-2x]=-2 \to y=-x$

$-1\le -2x<0 \to [-2x]=-1 \to y=-2$

$0\le -2x<1 \to [-2x]=0 \to y=-1$

$1\le -2x<2 \to [-2x]=1 \to y=0$



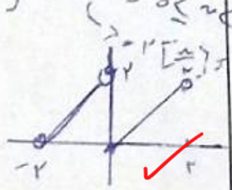
ب) $-2<x<2 \to -1<\frac{x}{2}<1$

$-1<\frac{x}{2}<0 \to [\frac{x}{2}]=-1 \to -2[\frac{x}{2}]+2$

$0\le\frac{x}{2}<1 \to [\frac{x}{2}]=0 \to 0\le x<2$

$(-2,0)+2=(0,2)$

$[0,2)-0=[0,2)$



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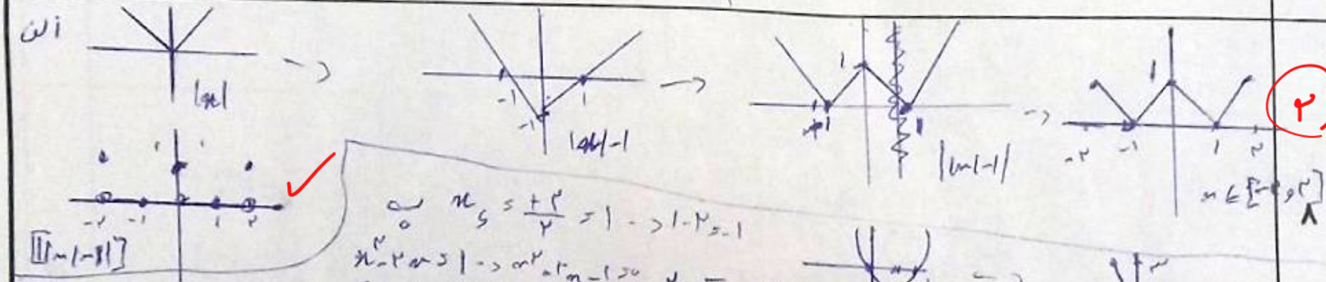
الف)



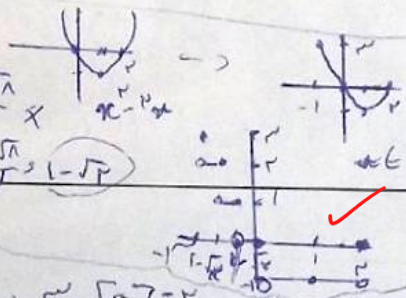
$x_s=\frac{+2}{2}=1 \to 1-2=-1$

$x^2-2x=1 \to x^2-2x-1=0$

$\Delta=4-4(1)(-1)=8$, $\frac{2\pm\sqrt{8}}{2}$, $\frac{2+\sqrt{8}}{2}$, $\frac{2-\sqrt{8}}{2}=1-\sqrt{2}$

$x^2-2x=2 \to x^2-2x-2=0$

$\Delta=4-4(1)(-2)=12$, $\frac{2\pm\sqrt{12}}{2}$, $\frac{2+\sqrt{12}}{2}$, $\frac{2-\sqrt{12}}{2}=1-\sqrt{3}$



$x\in[-1,3]$

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$[a]+x+[b]=4.2 \to [a]+[b]=4$

$[b]+y-[a]=2.4 \to [b]-[a]=2 \to 3-[a]=2 \to 3-2=[a] \to 1=[a]$

$2[b]=6 \to [b]=3$

$[a]+x+[b]+y=1+0.2+3+0.4=4.6$

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۱۰

$[(1+\sqrt{2})^6]+x+[(1-\sqrt{2})^6]+y=198 \to [(1+\sqrt{2})^6]+[(1-\sqrt{2})^6]=197$

$1-\sqrt{2}\in[0,1)$ $\quad [0,1)^6\in[0,1) \Rightarrow [(1+\sqrt{2})^6]+[[0,1)]=197=[(1+\sqrt{2})^6]+0=197$

$[(1+\sqrt{2})^6]=197$

۲

۲- ب)

$$\left|\frac{x-۲}{۲x+۱}\right| > ۱ \qquad x \neq \frac{-۱}{۲}$$

$$|x-۲| > |۲x+۱| \xrightarrow{\text{توان ۲}} x^{۲}-۴x+۴ > ۴x^{۲}+۴x+۱$$

$$۳x^{۲}+۸x-۳<۰ \longrightarrow$$

	-۳		$\frac{۱}{۳}$	
$+$		$-$		$+$

$$\left(-۳,\frac{۱}{۳}\right)-\left\{-\frac{۱}{۲}\right\}$$