نام و نام خانوادگی ...... پاسخنامه تشریحی تکلیف شماره ...... کلاس ......

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$$\frac{1}{x-1}=|x-1| \begin{cases} x>1 \Rightarrow \frac{1}{x-1}=x-1 \Rightarrow 1=x^2-2x+1 \Rightarrow x^2-2x=0 \begin{cases} x=0 \; ✗ \\ x=2 \; ✓ \end{cases} \\ x<1 \Rightarrow \frac{1}{x-1}=1-x \Rightarrow 1=-x^2+2x-1 \Rightarrow x^2-2x+2=0 \end{cases}$$

در اعداد مثبت

$$\Delta=b^2-4ac \Rightarrow (-2)^2-4(1)(2)=4-8=-4 \; ✗$$

ریشه ندارد

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الف) $-2<\frac{2x-1}{x}<2 \Rightarrow -2<2-\frac{1}{x}<2 \Rightarrow -4<-\frac{1}{x}<0 \Rightarrow 4>\frac{1}{x}>0$

$\frac{1}{4}<x<\infty \Rightarrow \frac{1}{4}<x$

$\frac{x-2}{2x+1}<-1 \Rightarrow \frac{1}{2}-\frac{2.5}{2x+1}<-1 \Rightarrow \frac{2.5}{2x+1}>\frac{3}{2} \Rightarrow 2x+1<\frac{15}{4} \Rightarrow x<\frac{11}{8}$

ب) $\frac{x-2}{2x+1}>-1 \Rightarrow \frac{x+0.5-2.5}{2x+1}>-1 \Rightarrow \frac{1}{2}-\frac{2.5}{2x+1}>-1 \Rightarrow \frac{-2.5}{2x+1}>\frac{1}{2} \Rightarrow \frac{2.5}{2x+1}<-\frac{1}{2}$

$\Rightarrow \frac{2x+1}{2.5}>-2 \Rightarrow 2x+1>-5 \Rightarrow 2x>-6 \Rightarrow x>-3$

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$$|x+1|-|2x|+|x-2| \begin{cases} x+1-2x+x-2=-1 & x\geq 2 \\ x+1-2x-x+2=-2x+3 & 2>x>0 \Rightarrow (-1,3) \\ x+1+2x-x+2=2x+3 & 0\geq x>-1 \Rightarrow (1,3] \\ -x-1+2x-x+2=1 & x\leq -1 \end{cases}$$

$R_f=R_1+R_2+R_3+R_4=\{-1\}\cup(-1,3)\cup(1,3]\cup\{1\}=[-1,3]$

$3+-1=2$

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$x^2<x+6 \qquad x^2-x-6<0 \qquad \Delta=1-4(1)(-6)=25$

$\frac{1\pm\sqrt{25}}{2} \begin{cases} 3 \\ -2 \end{cases} \qquad + \;|^{-2}\; - \;|^{3}\; + \Rightarrow x\in(-2,3)$

$x^2>-x-6 \Rightarrow x^2+x+6>0 \qquad \Delta=b^2-4ac=1-4(1)(6)=-23 \; ✗$

$\frac{3+(-2)}{2}=\frac{1}{2} \qquad \frac{3-(-2)}{2}=2\frac{1}{2} \qquad |x-\frac{1}{2}|<2\frac{1}{2} \Rightarrow a=\frac{1}{2}$

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تابع جدید $=|\frac{1}{2}x+2|-1 \qquad |\frac{1}{2}x+2|-1=|\frac{1}{2}x|-2 \Rightarrow |\frac{1}{2}x+2|=|\frac{1}{2}x|-1$

$$|\frac{1}{2}x+2|-|\frac{1}{2}x|=-1 \begin{cases} \frac{1}{2}x+2-\frac{1}{2}x=-1 \Rightarrow 2=-1 \; ✗ & x\geq 0 \\ \frac{1}{2}x+2+\frac{1}{2}x=-1 \Rightarrow x+2=-1 \Rightarrow x=-3 & 0>x>-4 \\ -\frac{1}{2}x-2+\frac{1}{2}x=-1 \Rightarrow -2=-1 \; ✗ & x\leq -4 \end{cases}$$

$-4 \qquad 0$

$-3 \Rightarrow |-\frac{3}{2}+\frac{4}{2}|-|-\frac{3}{2}|=-1$

$\frac{1}{2}-\frac{3}{2}=-1 \; ✓$

Scanned with CamScanner

الف) $|1-x|-|x+4| = \begin{cases} -1+x-x-4=-5 & x\geq 1 \\ 1-x-x-4=-2x-3 & 1>x\geq -4 \\ 1-x+x+4=5 & x\leq -4 \end{cases}$

$[R_f]=[R_1+R_2+R_3]$

$[\{-5\}\cup(-5,5)\cup\{5\}] = [[-5,5]] = \{-5,-4,-3,-2,-1,0,1,2,3,4,5\}$ ۶

ب) $\frac{1}{x}-[\frac{x}{x}+\frac{1}{x}] = \frac{1}{x}-[\frac{1}{x}]-1 = [0,1)-1 = [-1,0)$

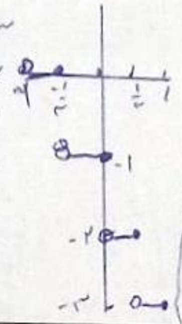
الف) $-1 < x \leq 1 \to -2 < -2x < 2$

$-2 < -2x < -1 \to [-2x] = -2 \to y = -3$

$-1 \leq -2x < 0 \to [-2x] = -1 \to y = -2$

$0 \leq -2x < 1 \to [-2x] = 0 \to y = -1$

$1 \leq -2x < 2 \to [-2x] = 1 \to y = 0$



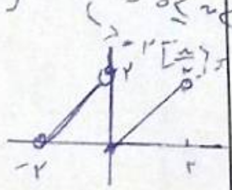
ب) $-2 < x < 2 \to -1 < \frac{x}{2} < 1$

$-1 < \frac{x}{2} < 0 \to [\frac{x}{2}] = -1 \to -2[\frac{x}{2}] = +2 \to -2 < x < 0$

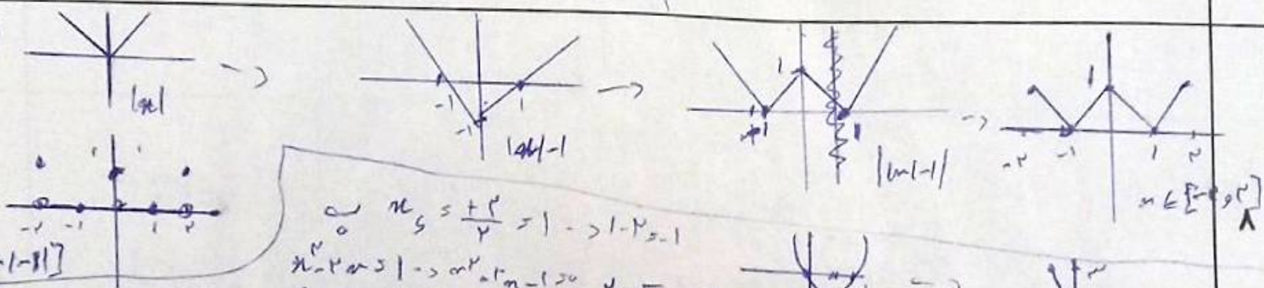
$0 \leq \frac{x}{2} < 1 \to [\frac{x}{2}] = 0 \to 0 \leq x < 2 \to -2[\frac{x}{2}] = 0$ ۷

$(-2,0)+2 = (0,2]$

$[0,2)-0 = [0,2)$



الف)



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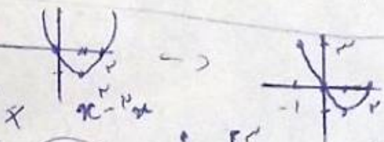
$x_s = \frac{\pm 2}{2} = 1 \to 1-2 = -1$

$x^2-2x = 1 \to x^2-2x-1 = 0$

$\Delta = 4-4(1)(-1) = 8$ $\quad \frac{2\pm\sqrt{8}}{2}$ $\quad \frac{2+\sqrt{8}}{2}$ ✗ $\quad \frac{2-\sqrt{8}}{2} = 1-\sqrt{2}$

$x^2-2x = 2 \to x^2-2x-2 = 0$

$\Delta = 4-4(1)(-2) = 12$ $\quad \frac{2\pm\sqrt{12}}{2}$ $\quad \frac{2+\sqrt{12}}{2}$ ✗ $\quad \frac{2-\sqrt{12}}{2} = 1-\sqrt{3}$



$[a]+x+[b] = 4/2 \to [a]+[b] = 4$

$[b]+y-[a] = 2/4 \to [b]-[a] = 2 \to 3-[a] = 2 \to 3-2 = [a] \to 1 = [a]$

$2[b] = 6 \to [b] = 3$ ۹

$[a]+x+[b]+y = 1+0/2+3+0/4 = 4/6$

$[(1+\sqrt{2})^6]+x+[(1-\sqrt{2})^6]+y = 198 \to [(1+\sqrt{2})^6]+[(1-\sqrt{2})^6] = 197$

$1-\sqrt{2}\in[0,1)$ $\quad [0,1)^6\in[0,1) \Rightarrow [(1+\sqrt{2})^6]+[[0,1)] = 197 = [(1+\sqrt{2})^6]+0 = 197$ ۱۰

$[(1+\sqrt{2})^6] = 197$