

۱- $\frac{1}{x-1} = |x-1| \xrightarrow{x-1=t} \frac{1}{t} = |||$

$\begin{cases} t > 0 \Rightarrow t^2 = 1 \Rightarrow t = 1 \Rightarrow x = 2 \\ t < 0 \Rightarrow -t^2 = 1 \Rightarrow \text{جواب ندارد} \end{cases}$

$t = x-1 = 1 \Rightarrow x = 2$

۲- الف) $|\frac{x-1}{x}| < 2 \rightarrow -2 < \frac{x-1}{x} < 2 \rightarrow -2 < \frac{x-1}{x} \Rightarrow 0 < \frac{x-1}{x} \Rightarrow \frac{x-1}{x} < 2 \Rightarrow \frac{x-1}{x} - 2 < 0 \Rightarrow \frac{-1}{x} < 0 \Rightarrow \frac{1}{x} > 0 \Rightarrow x > 0$

$\Rightarrow (\frac{1}{x}, +\infty)$

ب) $|\frac{x-2}{x+1}| > 1 \Rightarrow \frac{x-2}{x+1} > 1 \Rightarrow \frac{x-2}{x+1} - 1 > 0 \Rightarrow \frac{-x-3}{x+1} > 0 \Rightarrow \frac{-x-3}{x+1} < 0 \Rightarrow \frac{x+3}{x+1} < 0$

$\Rightarrow (-3, -1) \cup (-1, -\frac{1}{3})$

۳- $x^2 < |x+y|$ $|x-\alpha| < \beta$

$\Rightarrow \text{جواب: } \alpha, \beta$

$x^2 < |x+y| \Rightarrow \begin{cases} x^2 < x+y \Rightarrow x^2 - x - y < 0 \Rightarrow (x-3)(x+2) < 0 \Rightarrow \frac{-2}{x} < \frac{3}{x} \\ x^2 > x+y \Rightarrow x^2 - x - y > 0 \Rightarrow \text{جواب ندارد} \end{cases}$

$\Rightarrow -2 < x < 3 \Rightarrow -2, 0 < x-0, 0 < 3, 0 \Rightarrow |x-0, 0| < 2, 0 \Rightarrow \begin{cases} B = (-2, 0) \\ \alpha = (0, 3) \end{cases}$

۴- $y = |x+1| - |x-2| \Rightarrow \begin{cases} -1 & x \geq 2 \\ 3-2x & 0 \leq x < 2 \\ x+3 & -1 \leq x < 0 \\ 1 & x < -1 \end{cases}$

$\Rightarrow 3-1 = 2$

۵- $y = |\frac{1}{x}x| - 2 \xrightarrow{x \rightarrow x+\epsilon} y = |\frac{1}{x}(x+\epsilon)| - 2 + 1 = |\frac{1}{x}x + \frac{\epsilon}{x}| - 1$

$y_{\text{new}} = y_{\text{first}} \Rightarrow |\frac{1}{x}x + 2| - 1 = |\frac{1}{x}x| - 2 \Rightarrow |\frac{1}{x}x + 2| = |\frac{1}{x}x| - 1$

$\begin{cases} x > 0 \Rightarrow y = -1 \times \\ -\epsilon < x < 0 \Rightarrow \frac{1}{x}x + 2 = -\frac{1}{x}x \Rightarrow x = -3 \checkmark \\ x < -\epsilon \Rightarrow -y = -1 \times \end{cases}$

۶- $[|1-x| - |x+\epsilon|] = f(x) \Rightarrow \begin{cases} 1-x-1 & x \geq 1 \\ -2-2x & -\epsilon \leq x < 1 \\ 1-x-1 & x < -\epsilon \end{cases}$

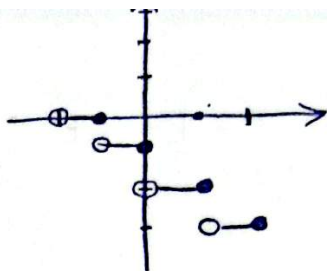
$\text{مجموعه: } \mathbb{R} = \{\pm 0, \pm \epsilon, \pm 1, \pm 2, \pm 3, \pm 4, \dots\}$

۷- $f(x) = \frac{1}{x} - [1 + \frac{1}{x}] = \frac{1}{x} - [\frac{1}{x}] - 1 \Rightarrow \lim_{x \rightarrow +\infty} \frac{1}{x} - [\frac{1}{x}] - 1 = 0$

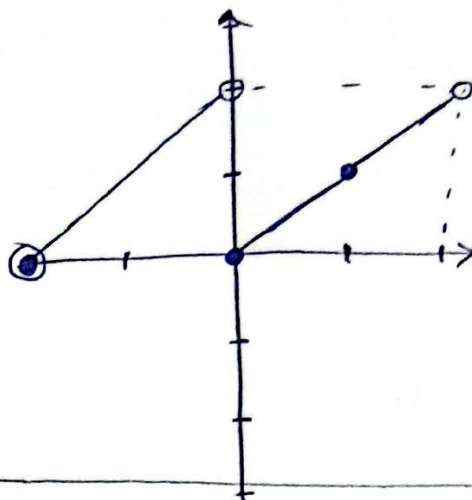
$\lim_{x \rightarrow -\infty} \frac{1}{x} - [\frac{1}{x}] - 1 = 0$

$\text{مجموعه: } R = [0, 1, 0) = (-1, 0)$

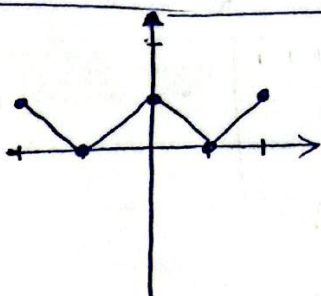
$$y = [-x] - 1 \quad x \in (-1, 1)$$



$$y = x - \left\lfloor \frac{x}{2} \right\rfloor \quad x \in (-2, 2)$$

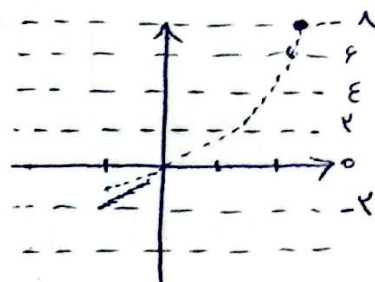
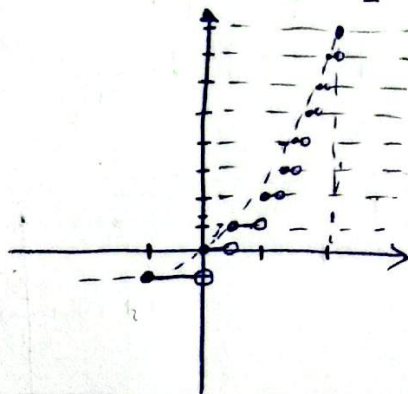


$$y = [||x| - 1|] \quad x \in [-2, 2]$$



$$y = [x^2 + x]$$

$$x \in [-1, 2]$$



$$\begin{aligned} b - [a] = 2, 8 & \quad a + [b] = 8, 2 \Rightarrow [a] + [b] = 8 \\ a + b = 1, 2 + 3, 8 = 8, 6 & \quad -[a] + [b] = 2 \\ [b] = 2 \Rightarrow b = 3, 8 & \quad [a] = 1 \Rightarrow a = 1, 2 \end{aligned}$$

$$(1 + \sqrt{2})^4 + (1 - \sqrt{2})^4 = 19 \Rightarrow (1 + \sqrt{2})^4 = 19 - \underbrace{(1 - \sqrt{2})^4}_{\frac{-8}{10}} \Rightarrow (1 + \sqrt{2})^4 = 19, \dots$$

$$\Rightarrow [(1 + \sqrt{2})^4] = 19$$