

$$BE^2 = 9x^2 + x^2 = 10x^2 \rightarrow BE = \sqrt{10}x \quad AC^2 = 18x^2 \rightarrow AC = 3\sqrt{2}x$$

$$\left. \begin{array}{l} \hat{C}_1 = \hat{A}_1 \\ \hat{E}_1 = \hat{B}_1 \\ F_1 = F_p \end{array} \right\} \Rightarrow \triangle AEF \sim \triangle BFC \Rightarrow \begin{cases} AF = \frac{1}{3} FC \\ EF = \frac{1}{3} BF \end{cases}$$

$$k = 3 \rightarrow \text{نسبت}$$

$$\rightarrow EF = \sqrt{10}x \rightarrow y = \frac{\sqrt{10}}{3}x$$

$$\rightarrow FZ = 3\sqrt{2}x \rightarrow Z = \frac{3\sqrt{2}}{3}x$$

$$\frac{EF}{AF} = \frac{y}{z} = \frac{\frac{\sqrt{10}}{3}x}{\frac{3\sqrt{2}}{3}x} = \frac{\sqrt{10}}{3\sqrt{2}} = \frac{\sqrt{5}}{3}$$

$$\left. \begin{array}{l} \hat{A} = \hat{A} \\ \hat{E} = \hat{C} \end{array} \right\} \Rightarrow \triangle AED \sim \triangle ACB \Rightarrow D_1 = \hat{B}$$

$$\frac{1}{x+1} = \frac{x}{2} \rightarrow x^2 + x = 2 \rightarrow x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0 \rightarrow x = 1$$

$$\left. \begin{array}{l} C_1 = D_1 \\ F_1 = E_1 \\ H_1 = H_p \end{array} \right\} \Rightarrow \triangle BEH_p \sim \triangle CFH_1$$

$$3y = 2x$$

$$x = \frac{3}{2}y = 0.75y$$

$$\frac{AE}{AC} = \frac{DE}{BC} \rightarrow \frac{0.75x}{5} = \frac{0.75y}{3+y} \rightarrow 0.75x = 3+y \rightarrow 0.75y = 3+y$$

$$BC = 3+y = \frac{2.25 + 3.75}{1} = \frac{6}{1} = 6$$

$$BH^2 = 1 \times 4 = 4 \Rightarrow BH = 1$$

$$BA^2 = 1^2 + 4 = 5 \rightarrow BA = \sqrt{5}$$

$$BC^2 = 2^2 + 4 = 8 \rightarrow BC = \sqrt{8} = 2\sqrt{2}$$

$$\left. \begin{array}{l} A_1 = C_1 \\ D_1 = E_1 \\ F_1 = F_p \end{array} \right\} \Rightarrow \triangle ABF \sim \triangle CEF$$

$$\rightarrow \frac{1+x}{2} = \frac{y}{2} \rightarrow x^2 + 1x = y^2$$

$$x^2 + 1x - y^2 = 0$$

$$(x+1)(x-y) = 0 \rightarrow x = y$$

$$AF = 1+y = 2$$

$\hat{B} = \hat{C} = 90^\circ \Rightarrow \hat{B}_1 = \hat{B}_2 = \hat{C}_1 = \hat{C}_2 = 90^\circ$: بنابراین
 $\hat{H} = 90^\circ \Rightarrow \hat{D}_1 = \hat{D}_2 = \hat{E}_1 = \hat{E}_2 = 90^\circ$ پس مثلث BDF قائم الزاویه متساوی الساقین است
 بنابراین: $DH^2 = BH \times HE = 4 \times 1 = 4 \Rightarrow DH = 2$

$HD \parallel BC \Rightarrow \frac{x + 11}{14 + 11} = \frac{2}{11} \Rightarrow 11x + 44 = 14x + 22 \Rightarrow 3x = 22 \Rightarrow x = \frac{22}{3}$

$AH^2 = 3x \times x = 3x^2 \Rightarrow AH = \sqrt{3}x \xrightarrow{AH=CG} CG = \sqrt{3}x$
 $AB^2 = 3x \times 4x = 12x^2 \Rightarrow AB = 2\sqrt{3}x$
 $AD^2 = x \times 4x = 4x^2 \Rightarrow AD = 2x$

$\triangle DHE \sim \triangle ADH \Rightarrow \frac{x}{\sqrt{3}x} = \frac{y}{2x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{y}{2} \Rightarrow y = \frac{2}{\sqrt{3}}$

$S_{DHE} = \frac{\frac{x}{\sqrt{3}} \times x}{2} = \frac{x^2}{2\sqrt{3}}$

$S_{ABCD} = 2\sqrt{3}x \times 2x = 4\sqrt{3}x^2 \xrightarrow{\frac{4\sqrt{3}x^2}{\frac{x^2}{2\sqrt{3}}}} = 1 \times 24 = 24$

$\frac{S_{BMN}}{S_{ABC}} = \frac{BM \times BN \times \cos \theta}{BC \times AB \times \cos \theta} = \frac{BM}{AB} \times \frac{BN}{AC} = \frac{1}{3}$

$\Rightarrow \frac{BM}{AB} = \frac{1}{9} \Rightarrow \frac{BM}{AM} = \frac{1}{8} = 1, 2$

با توجه به زوایا، دو مثلث درون شکل متشابه اند و نسبت تناسبات آنها نیز ۴ است.

$4y + x = 4x \Rightarrow 4y = 3x \Rightarrow y = \frac{3}{4}x$

$x^2 + y^2 = 1 \Rightarrow x^2 + \frac{9}{16}x^2 = 1 \xrightarrow{\times 16} 16x^2 + 9x^2 = 16 \Rightarrow 25x^2 = 16 \Rightarrow x^2 = \frac{16}{25} \Rightarrow x = \frac{4}{5}$

$y = \frac{3}{4} \times \frac{4}{5} = \frac{3}{5}$

$S_{\text{مربع}} = 4x \times 4x = \frac{4 \times 4}{25} = \frac{16}{25} = 10, 24$

$GD = \frac{1}{3}BD$ — محل هر سی سه میانه است

در مثلث AGC، AF و GD، میانه اند و نقطه F محل هر سی میانه اند.

$FD = \frac{1}{3}GD \Rightarrow FD = \frac{1}{3} \times \frac{1}{3}BD \Rightarrow \frac{BD}{FD} = 9$