

$$BC = CD = AD = AB = 1 \Rightarrow \frac{BC}{AE} = \frac{1}{x} = \frac{1}{x}$$

$$\frac{FC}{FA} = \frac{1}{x} \Rightarrow \frac{FC}{1-x} = \frac{1}{x} \Rightarrow FC = \frac{1-x}{x}$$

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$$\triangle AEF \sim \triangle BFC$$

$$\Rightarrow \frac{AE}{BF} = \frac{AF}{BC} = \frac{EF}{FC}$$

$$\Rightarrow \frac{x}{x} = \frac{1-x}{1} = \frac{EF}{FC}$$

$$\Rightarrow 1 = \frac{EF}{FC} \Rightarrow EF = FC$$

$$\triangle ABE \sim \triangle BCF \Rightarrow \frac{AE}{BF} = \frac{AB}{BC} = \frac{BE}{CF}$$

$$\Rightarrow \frac{x}{x} = \frac{1}{1} = \frac{BE}{CF} \Rightarrow BE = CF$$

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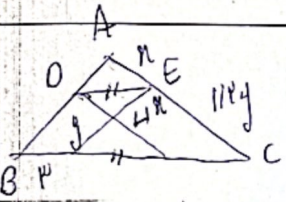
$$\Rightarrow \frac{BE}{CF} = \frac{1}{1} \Rightarrow BE = CF$$

$$\frac{AD}{AB} = \frac{ED}{BC} = \frac{AE}{AC} \Rightarrow \frac{1}{1} = \frac{ED}{1} = \frac{x}{\sqrt{2}}$$

$$\Rightarrow ED = x$$

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$$\Rightarrow x = \frac{\sqrt{2}}{2}$$



$$\triangle ADE \sim \triangle ABC \Rightarrow \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$

$$\Rightarrow \frac{x}{1} = \frac{x}{1} = \frac{DE}{1} \Rightarrow DE = x$$

$$\Rightarrow \frac{DE}{BC} = \frac{x}{1} = \frac{x}{\sqrt{2}}$$

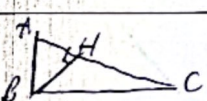
$$\Rightarrow x = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \frac{DE}{BC} = \frac{x}{1} = \frac{x}{\sqrt{2}}$$

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$$\Rightarrow x = \frac{\sqrt{2}}{2}$$



$$\triangle ABH \sim \triangle ACH \Rightarrow \frac{AB}{AC} = \frac{BH}{CH}$$

$$\Rightarrow \frac{1}{1} = \frac{BH}{CH} \Rightarrow BH = CH$$

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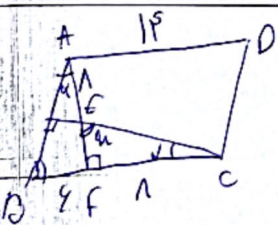
$$\Rightarrow \frac{BH}{CH} = \frac{1}{1} \Rightarrow BH = CH$$

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$$\Rightarrow \vec{ED} = \vec{DB}$$

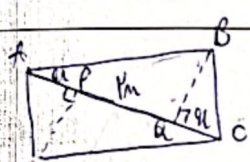
$$\Rightarrow \frac{m+1}{2/2+1} = \frac{1}{2/2} \Rightarrow \vec{ED} + \vec{DB} = 2/2 \text{ or } \parallel 2/2$$

$$\Rightarrow 2/2 = \parallel (2/2 - \vec{E})$$

$$\Rightarrow \vec{a} = \frac{11 \times 1/2}{1/2} = \frac{44}{1} \Rightarrow m = 4/4 \checkmark$$

(2)

6



$$ADP, ABQ = \text{similar} \Rightarrow \vec{A} = \vec{A} \Rightarrow \vec{ABQ} = \vec{ADP}$$

$$\Rightarrow \frac{AB}{AD} = \frac{AP}{AQ} \Rightarrow AQ = AP + PQ = m + m = 2m$$

$$\Rightarrow \frac{AB}{AD} = \frac{m}{2m} = \frac{1}{2} \Rightarrow \delta = AB \times AD \Rightarrow AD = 2m \Rightarrow \delta = 2m^2 \Rightarrow \delta_{ABQ} = \frac{1}{2} \delta_{ADP}$$

$$\Rightarrow \frac{BQ}{DP} = \frac{AB}{AD} = \frac{1}{2} \Rightarrow BQ = h \Rightarrow DP = 2h \Rightarrow \delta_{ABQ} = \frac{1}{2} \delta_{ADP} \Rightarrow \delta_{ADP} = \frac{1}{2} \delta_{ABQ}$$

$$\Rightarrow \delta_{ABQ} = \delta_{ADP} \quad \delta_{ABQ} = 4 \quad \frac{\delta_{big}}{\delta_{small}} = \frac{6}{1} = 6 \Rightarrow 4 \times 6 = 24 \checkmark$$

(2)

1

$$\frac{BM}{AM} = u \Rightarrow BM = uAM \Rightarrow AB = AM + BM = AM(1+u) \Rightarrow \frac{BM}{AB} = \frac{u}{1+u} \Rightarrow \frac{BN}{NC} = \frac{u}{1} \Rightarrow \frac{BN}{BC} = \frac{u}{1+u}$$

$$\Rightarrow \frac{\delta_{BMN}}{\delta_{ABC}} = \frac{BM}{AB} \times \frac{BN}{BC} \Rightarrow \frac{\delta_{BMN}}{\delta_{ABC}} = \frac{u}{1+u} \times \frac{u}{1} \Rightarrow \frac{u}{1+u} \times \frac{u}{1} = \frac{1}{u} \Rightarrow 4m = 1 + 2m \Rightarrow m = \frac{1}{2}$$

$$\Rightarrow \frac{BM}{AM} = \frac{1}{2} \checkmark$$

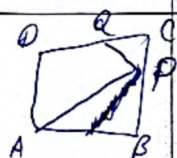
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$$AP = 5 \quad PQ = 1 \Rightarrow DQ = PQ = 1 \Rightarrow \delta = \vec{PQ} \Rightarrow AQ = 5$$

$$\rightarrow PP = 1 \quad PP \parallel PQ \Rightarrow AP = AP + PP \Rightarrow \vec{P} = \vec{Q}$$

$$AP = 2 \Rightarrow AC = \sqrt{5} \Rightarrow AC = \sqrt{5}$$

$$\Rightarrow (AC)^2 = (AP)^2 + \delta^2 \Rightarrow (\sqrt{5})^2 = 2^2 + \delta^2 \Rightarrow 5 = 4 + \delta^2 \Rightarrow \delta^2 = 1 \Rightarrow \delta = 1$$



For this problem, PQ = 1 is given

9

$$G = \sqrt{3} \times \vec{P} \Rightarrow GE = EC \Rightarrow \frac{AG}{GM} = \vec{P} \Rightarrow \vec{P} \quad CG = GE = \frac{EC}{\vec{P}}$$

$$\Rightarrow B, G, C = \text{collinear}$$

$$A, F, E = \text{collinear}$$

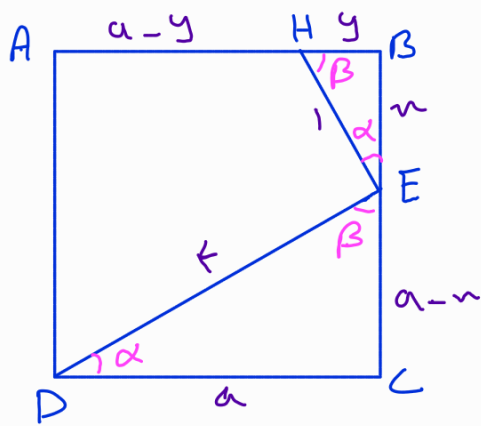
$$B, F, D = \text{collinear}$$

$$\Rightarrow \frac{BG}{GM} = \vec{P} \Rightarrow \frac{BD}{FD} = \frac{BG}{GF} \Rightarrow \frac{BG}{GC} = \vec{P} \Rightarrow GC = GE$$

$$\Rightarrow GE = GF \Rightarrow \frac{BG}{GF} = \vec{P} \Rightarrow \frac{BD}{FD} = \vec{P}$$

(2)

10



$$\triangle EBH \sim \triangle DCE \rightarrow \frac{y}{a-n} = \frac{n}{a} = \frac{1}{k} \begin{cases} a = kn \\ a-n = ky \\ n = \frac{k}{k+1}y \end{cases}$$

$$\triangle DCE \xrightarrow{\text{پیتاگورس}} (k)^2 = (a-n)^2 + (a)^2 \rightarrow \begin{cases} n = \frac{k}{k+1}y \\ a = \frac{1}{k+1}y \end{cases}$$

$$S_{\square} = a^2 = \boxed{\frac{256}{25}}$$

$$\text{محل برخورد ضلع ها } G \rightarrow BG = 2n \quad GD = n \rightarrow BD = 3n$$

از A به G وصل کنیم، AE، GD ضلع ها مثلث AGC مرئوسه :

$$\begin{aligned} GF &= \frac{2}{3}n \\ FD &= \frac{1}{3}n \end{aligned} \Rightarrow \frac{BD}{FD} = \frac{3n}{\frac{1}{3}n} = \boxed{9}$$