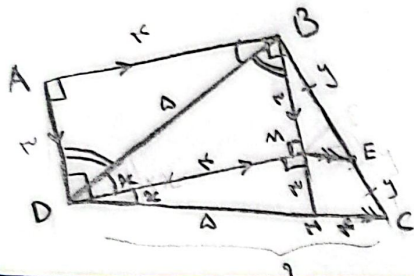


$$\frac{S_{BCE}}{S_{BDE}} \quad \frac{A}{12} = \frac{DE}{BC} \rightarrow \Delta BCE = 12 \Delta DE$$

$$\rightarrow = \frac{\frac{1}{2} \times BE \times BC \times \sin \alpha}{\frac{1}{2} \times DE \times BE \times \sin \alpha} = \frac{12}{\frac{A}{12}} = \frac{12}{\Delta}$$

(1)

DM

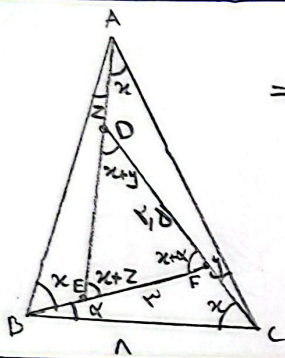


$$\frac{BE}{BC} = \frac{ME}{HC} \rightarrow \frac{5}{12} = \frac{ME}{4} \rightarrow \boxed{ME = 2}$$

HC و ME

طبق عکس قضیه تالس با هم موازی اند.

(2)



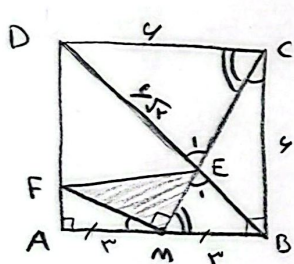
$$\Rightarrow \begin{cases} \hat{A} = \hat{E} \\ \hat{B} = \hat{F} \\ \hat{C} = \hat{D} \end{cases}$$

$$\Delta ABC \sim \Delta DFE$$

(3) زاویه خارج یک راس = مجموع دو زاویه داخلی دیگر

$$\frac{DF}{FE} = \frac{BC}{AB} \rightarrow \frac{2\Delta}{3} = \frac{1}{AB} \rightarrow \boxed{AB = 6,6}$$

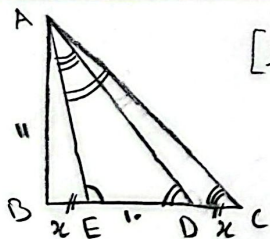
(4)



$$\Delta AMF \sim \Delta MBC \rightarrow \frac{6}{3} = \frac{3}{AF} \rightarrow \boxed{AF = \frac{3}{2}} \quad MC = \sqrt{3^2 + 9} = 3\sqrt{2}$$

$$[ز] \Delta DCE \sim \Delta MBE \rightarrow \frac{DC}{MB} = \frac{EC}{ME} \rightarrow \frac{6}{3} = \frac{EC}{ME} \rightarrow 2ME = EC \rightarrow 3ME = 3\sqrt{2} \rightarrow \boxed{ME = \sqrt{2}}$$

$$FM = \sqrt{9 + \frac{9}{4}} = \sqrt{\frac{45}{2}} = \frac{3\sqrt{10}}{2} \Rightarrow S_{FME} = \frac{\frac{3\sqrt{2}}{2} \times \sqrt{2}}{2} = \boxed{\frac{15}{4}}$$



$$[ز] \Delta AED \sim \Delta AEC$$

$$AE = \sqrt{11 + x^2}$$

$$\frac{AE}{EC} = \frac{ED}{AE}$$

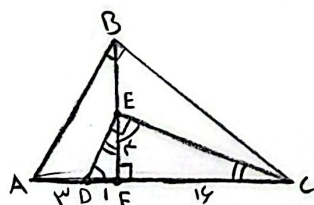
$$\rightarrow \frac{\sqrt{11 + x^2}}{1 + x} = \frac{1}{\sqrt{11 + x^2}} \rightarrow 11 + x^2 = 1 + 2x + x^2 \rightarrow 10 = 2x \rightarrow x = 5$$

DC = ?

$$x^2 - 10x + 11 = 0$$

$$\boxed{x = 7,3} \leftarrow (x - 7)(x - 3) = 0$$

(5)

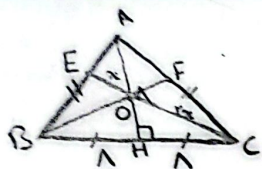


$$\Delta DFE \sim \Delta EFC \rightarrow \frac{EF}{DF} = \frac{CF}{EF} \rightarrow \frac{4}{1} = \frac{CF}{4} \rightarrow \boxed{CF = 16}$$

BC = ?

$$\Rightarrow BC^2 = CF \times CA \rightarrow \boxed{BC = \sqrt{16 \times 20} = 4\sqrt{10}}$$

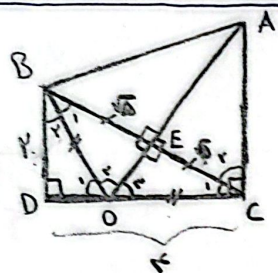
(6)



(V) میانه‌ها با نسبت ۲ و ۱ تقسیم می‌شوند. در مثلث $\triangle OHC$ $OH = \frac{\Delta}{\sqrt{3}} / AO = \frac{2 \times \Delta}{\sqrt{3}} \leftarrow OHC$

$$\rightarrow OC = \sqrt{\left(\frac{\Delta}{\sqrt{3}}\right)^2 + \Delta^2} = \sqrt{\frac{\Delta^2 + 3\Delta^2}{3}} = \sqrt{\frac{4\Delta^2}{3}} = \frac{2\Delta}{\sqrt{3}} = 2\Delta$$

$$EC = 2\Delta \rightarrow \left(\frac{\Delta}{\sqrt{3}} + \frac{\Delta}{\sqrt{3}}\right)\sqrt{3} = \frac{2\Delta}{\sqrt{3}}\sqrt{3} = 2\Delta$$



$$S_{ABE} = ?$$

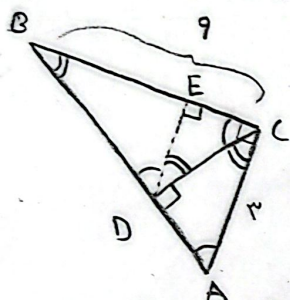
(A) در مثلث $\triangle BOC$ چون متساوی الساقین است و $\angle O = 90^\circ = \angle E$ میانه نیز است

$$\left. \begin{aligned} B_1 + B_2 + C_1 &= 90^\circ \\ B_1 &= C_1 \end{aligned} \right\} 2B_1 + B_2 = 90^\circ \rightarrow O_1 = B_1 + C_1 = 2B_1 \rightarrow O_1 + O_2 + O_3 = 180^\circ = 2B_1 + 2O_2$$

$$\rightarrow B_1 + O_2 = 90^\circ = 2B_1 + B_2 \rightarrow O_2 = B_1 + B_2$$

$$BC = \sqrt{4+16} = \sqrt{20} \rightarrow EB = EC = \sqrt{5} \xrightarrow{C_1=B_1} \triangle ACE \sim \triangle BDC \quad \frac{DC}{EA} = \frac{BD}{EC}$$

$$\rightarrow \frac{4}{EA} = \frac{2}{\sqrt{5}} \rightarrow EA = 2\sqrt{5} \Rightarrow S_{ABE} = \frac{\sqrt{5} \times 2\sqrt{5}}{2} = 5$$



$$AB = \sqrt{1+9} = \sqrt{10}$$

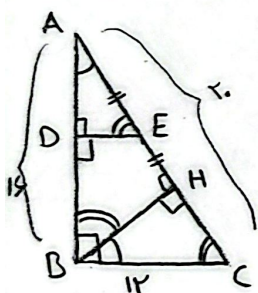
$$\triangle ABC \sim \triangle ADC \rightarrow \frac{CD}{BC} = \frac{CA}{AB}$$

$$\frac{CD}{9} = \frac{4}{\sqrt{10}} \rightarrow CD = \frac{36\sqrt{10}}{11}$$

$$BE = ? \quad (9)$$

$$\rightarrow \triangle ABC \sim \triangle EDC \rightarrow \frac{ED}{BC} = \frac{DC}{BA} \rightarrow \frac{ED}{9} = \frac{\frac{36\sqrt{10}}{11}}{\sqrt{10}} \rightarrow ED = \frac{36}{11} = 3\frac{3}{11}$$

$$\Rightarrow \triangle BED \sim \triangle ABC \Rightarrow \frac{EB}{BC} = \frac{ED}{CA} \rightarrow EB = \frac{9 \times 9 \times 4}{1 \times 11} = 29\frac{1}{11}$$



$$AC = \sqrt{1+9+4+9} = \sqrt{23} = \sqrt{23}$$

$$DE = ? \quad (10)$$

$$\triangle ABC \sim \triangle BHC \rightarrow \frac{HC}{BC} = \frac{BC}{AC} \rightarrow \frac{HC}{12} = \frac{12}{\sqrt{23}} \rightarrow HC = \frac{144}{\sqrt{23}}$$

$$HC = \frac{144}{\sqrt{23}}$$

$$AH = 2AE = 2 - \frac{144}{\sqrt{23}} = \frac{2\sqrt{23} - 144}{\sqrt{23}}$$

$$\rightarrow AE = \frac{2\sqrt{23} - 144}{2\sqrt{23}}$$

$$\Rightarrow \triangle ADE \sim \triangle ABC \rightarrow \frac{AE}{AC} = \frac{DE}{BC} \rightarrow \frac{\frac{2\sqrt{23} - 144}{2\sqrt{23}}}{\sqrt{23}} = \frac{DE}{12} \rightarrow DE = \frac{12(2\sqrt{23} - 144)}{23}$$