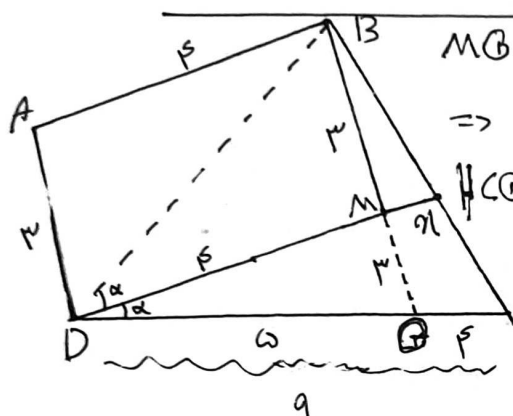


۳. در مثل ABC و DE برای هستن بین مثلت با هم ارتفاع هستن نسبت به دایره

نسبت متعکس دایره

$$\frac{DE}{BC} = \frac{S_{\triangle DEB}}{S_{\triangle BEC}} = \frac{\frac{1}{2} DE \times h}{\frac{1}{2} BC \times h} \Rightarrow \frac{\Delta}{12} \Rightarrow \frac{S_{\triangle BEC}}{S_{\triangle DEB}} = \frac{12}{\Delta} = 2,4$$



DM یا BM یا MG برابر است $\Rightarrow MG=3$

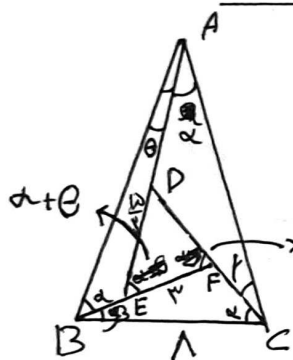
$$\Rightarrow DG = \sqrt{DM^2 + MG^2} = \sqrt{3^2 + 4^2} = 5$$

بر اساس مثلث قضیه تالس $\frac{BM}{MG} = \frac{BH}{HC}$ و خط MH موازی CG

برای این بین بر اساس قضیه تالس $\frac{BM}{BC} = \frac{MH}{CG} \Rightarrow \frac{3}{BC} = \frac{4}{5} \Rightarrow BC = \frac{15}{4}$

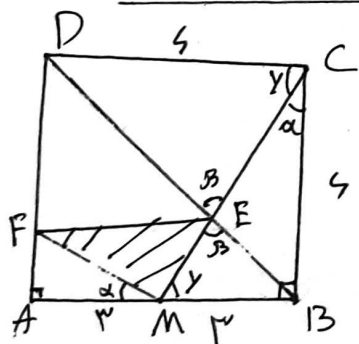
$$n = 2 \Leftarrow$$

با توجه به زاویه خارجی مثلث DEF با مثلث ABC مشابه می شود.



$$\left. \begin{array}{l} F = \alpha + \beta \\ E = \alpha + \gamma \\ D = \alpha + \gamma \end{array} \right\} \xrightarrow{\text{سزاوار}} \triangle DEF \sim \triangle BAC \Rightarrow \frac{EF}{AB} = \frac{DE}{BC}$$

$$\Rightarrow \frac{3}{AB} = \frac{2,4}{1} \Rightarrow AB = 9,5$$

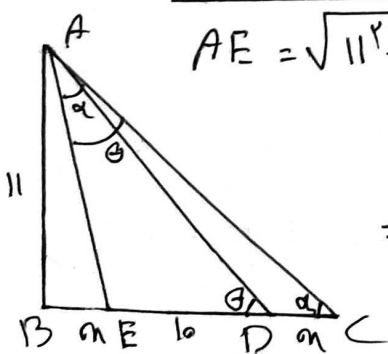


$$\alpha = \alpha \Rightarrow \angle A = \angle C \quad MC = \sqrt{4^2 + 3^2} = 5$$

$$\left. \begin{array}{l} \angle A = \angle B \\ \angle M = \angle F \end{array} \right\} \xrightarrow{\text{سزاوار}} \triangle AFM \sim \triangle BCG \Rightarrow \frac{AM}{BC} = \frac{FM}{MC}$$

$$\Rightarrow \frac{3}{4} = \frac{FM}{5} \Rightarrow FM = \frac{15}{4} \quad \left. \begin{array}{l} \angle E = \angle \gamma = \beta \\ \angle F = \angle \gamma = \alpha \end{array} \right\} \xrightarrow{\text{سزاوار}} \triangle PCE \sim \triangle MBE$$

$$\frac{MB}{PC} = \frac{ME}{CE} \Rightarrow \frac{3}{5} = \frac{ME}{5} \Rightarrow ME = 3 \quad \Rightarrow S_{FEM} = \frac{1}{2} \times 3 \times \frac{15}{4} = \frac{45}{8} = 5,625$$

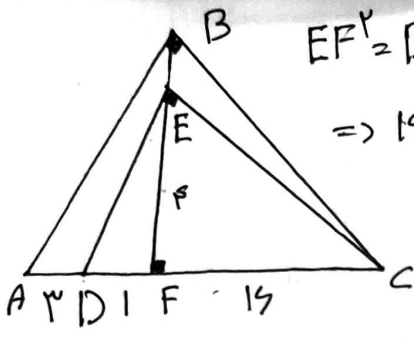


$$AE = \sqrt{11^2 + 10^2} \Rightarrow \alpha = \angle C = A, \quad \left. \begin{array}{l} \angle B = \angle D = A \\ \angle E = \angle \gamma \end{array} \right\} \xrightarrow{\text{سزاوار}} \triangle ADE \sim \triangle AEC$$

$$\Rightarrow \frac{ED}{AE} = \frac{AE}{EC} \Rightarrow \frac{10}{\sqrt{11^2 + 10^2}} = \frac{\sqrt{11^2 + 10^2}}{10 + n} \Rightarrow n^2 - 10n + 11 = 0$$

$$n = 3, 7 \Leftarrow \Rightarrow (n-3)(n-7) = 0$$

بر طبق رابطه طولی در مثل قائم الزامی ←

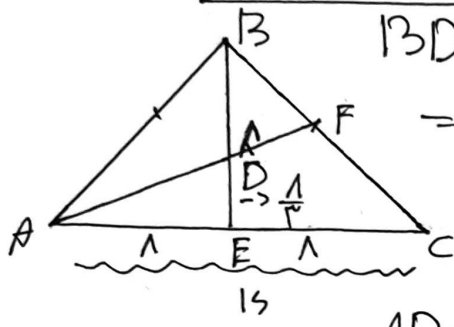


$$EF^2 = DF \times FC$$

$$\Rightarrow 15 = 1 \times FC \Rightarrow FC = 15$$

$$BC^2 = CF \times AC \Rightarrow 15 \times 30 = BC^2 \Rightarrow BC = 9\sqrt{2} = 12.72$$

9



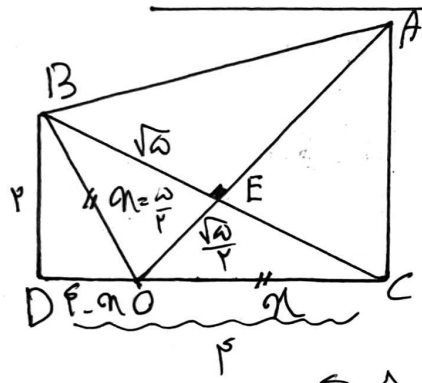
نسبت تناسل بین اضلاع مثلث برابر است یعنی $BD = 2DE$

$$\Rightarrow 2DE = 1 \Rightarrow DE = \frac{1}{2}$$

$$AD = \sqrt{AE^2 + DE^2} \Rightarrow \sqrt{5\frac{5}{9} + \frac{1}{4}} \Rightarrow \frac{\sqrt{50}}{2} = AD$$

1/2

$$\Rightarrow AD = 2FD \Rightarrow AF = \frac{\sqrt{50}}{2} \times \frac{2}{2} = \frac{1\sqrt{10}}{2} = 9\sqrt{10}$$



$$BO = \sqrt{DO^2 + BD^2} \Rightarrow \cancel{BO^2} = 9 + 15 + \cancel{BO^2} - 18$$

$$\Rightarrow BO = \frac{15}{2}$$

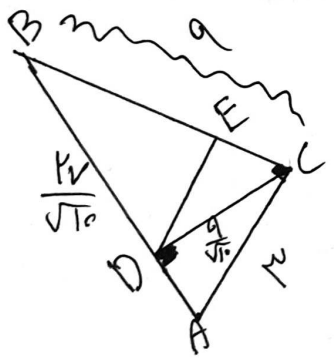
چون O مرکز دایره است و CG و BO در خط عمود
نیست پس AO مرکز دایره است

1

$$\Rightarrow BO = \sqrt{9 + 15} \Rightarrow \frac{\sqrt{24}}{2} = \sqrt{6} = BE = EC$$

$$EO = \frac{15}{2} = AO + EO \Rightarrow EO = \frac{\sqrt{6}}{2} \Rightarrow (\sqrt{6})^2 = \frac{\sqrt{6}}{2} \times AE \Rightarrow AE = 2\sqrt{6}$$

$$\Delta ABE = \frac{1}{2} \times \sqrt{6} \times 2\sqrt{6} = 6$$



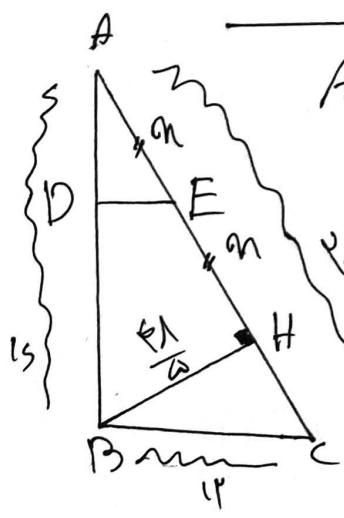
$$AB = \sqrt{AC^2 + BC^2} \Rightarrow \sqrt{9 + 11} = AB = 2\sqrt{10}$$

$$AC \times BC = AB \times CD \Rightarrow 3 \times 9 = 2\sqrt{10} \times CD \Rightarrow CD = \frac{9}{2\sqrt{10}}$$

9

$$BC^2 = AB \times BD \Rightarrow 11 = 2\sqrt{10} \times BD \Rightarrow BD = \frac{11}{2\sqrt{10}}$$

$$BD^2 = BE \times BC \Rightarrow \frac{121}{40} = 9 \times BE \Rightarrow BE = 1.1$$



$$AC = \sqrt{12^2 + 15^2} \Rightarrow AC = 20 \quad BH \times AC = AB \times BC$$

$$\Rightarrow BH \times 20 = 12 \times 15 \Rightarrow BH = \frac{9}{2}$$

$$20 = \frac{9}{2} + 15^2 = \left(\frac{9}{2}\right)^2 + 15^2 \Rightarrow 15 = 9.5$$

10

$$20 - 12 = 8 = HC \Rightarrow \frac{9.5}{20} = \frac{DE}{12} \Rightarrow DE = 5.7$$