عرشیا رحیمی تکلیف شماره ۲۳ ـ دهم سیر B

۱-

$$\log_{mn} m^2 n \to \log_{mn} m^2 + \log_{mn} n \to \frac{1}{\log_{m^2} mn} + \frac{1}{\log_{n} mn}$$

$$\to \frac{1}{\underbrace{\log_{m^2} m}_{\frac{1}{2}} + \underbrace{\log_{m^2} n}_{\frac{1}{2a}}} + \frac{1}{\underbrace{\log_{n} m}_{a} + \underbrace{\log_{n} n}_{1}}$$

$$\Rightarrow \frac{1}{\frac{1}{2}+\frac{1}{2a}} + \frac{1}{a+1} \Rightarrow \frac{2a+1}{a+1} \Rightarrow \left[\frac{2a+1}{a+1}\right] \Rightarrow \left[\frac{a+\cancel{a+1}}{\cancel{a+1}}\right]$$

$$\Big[\underbrace{\tfrac{a}{a+1}}_{\text{کوچکتر از یک}}\Big]+1 \Rightarrow 0+1=\boxed{1}$$

۲-

الف) $\frac{x}{\log_{\frac{1}{2}} x} \geq 0 \longrightarrow$

0	$\frac{1}{2}$
$+$	$-$

و $x>0$

$$\Rightarrow \boxed{D_f = (\tfrac{1}{2}, +\infty)}$$

ب) $x^2-x-2>0 \Rightarrow (x-2)(x+1)>0$

-1	2
$+$	$-$ $+$

$x^2-1 \geq 0 \to (x-1)(x+1) \geq 0$

-1	1
$+$	$-$ $+$

$\to D_f = (-\infty, -1) \cup (2, +\infty)$

۳-

$$2\log_{\underbrace{9}_{3^2}} a + \log_{a} \sqrt{9} = 2 \qquad (\sqrt{9} \to 3)$$

$$\to \log_{3} a + \log_{a} 3 = 2$$

$$\to \log_{3} a + \frac{1}{\log_{3} a} = 2 \xrightarrow{\log_3 a = t} t+\frac{1}{t}=2 \Rightarrow t^2-2t+1=0$$

$$\Rightarrow (t-1)^2=0$$

$$\Rightarrow t=1$$

$$\Rightarrow \log_{3} a = 1 \Rightarrow \boxed{a=3}$$

$(\log 5 - \log 3)x^2 + (\log 3^2)x - \log 5\times 3 = 0$ — ۴

$\rightarrow (\log\frac{10}{2} - 0/4)x^2 + (2\times 0/4)x - \log^{\frac{10}{2}} - \underbrace{\log^3}_{0/4} = 0$

$\rightarrow (1 - 0/3 - 0/4)x^2 + (0/8)x - (1 - 0/3) - 0/4 = 0$

$\Rightarrow 0/3x^2 + 0/8x - 1/1 = 0$

$\Rightarrow 3x^2 + 8x - 11 = 0 \rightarrow x^2 + 8x - 33 = 0$

$\Rightarrow (x+11)(x-3) = 0 \quad x = \frac{3}{3} = 1, \quad x = -\frac{11}{3}$

$\Rightarrow \left|1 + \frac{11}{3}\right| = \boxed{\frac{14}{3}}$

— ۵

$\log_{12}^{10} \rightarrow \frac{\log_2^{10}}{\log_2^{12}} \Rightarrow \frac{\log_2^5 + \log_2^2}{\log_2^3 + \log_2^4} = \frac{[illegible]}{[illegible]} = \boxed{[illegible]}$

— ۶

$\log_{15}^{6} \rightarrow \frac{\log_3^6}{\log_3^{15}} = \frac{\log_3^2 + \log_3^3}{\log_3^3 + \log_3^5} = \frac{1 + \frac{10}{16}}{1 + \frac{11}{5}}$

$\frac{26\times 5}{16\times 16} = \boxed{\frac{65}{128}}$

— ۷

$\log_4^{12} = \log_4^{3\times 4} \rightarrow \underbrace{\log_4^4}_{1} + \frac{1}{2}\log_2^3$

$\log_8^{18} = \log_8^{9\times 2} \rightarrow \log_{2^3}^{3^2} + \log_8^2 = m \Rightarrow \frac{2}{3}\log_2^3 + \frac{1}{3} = m$

$\Rightarrow \log_2^3 = \frac{3m-1}{2}$

$\Rightarrow 1 + \frac{3m-1}{4} = \boxed{\frac{3m+3}{4}}$

۸- از دو طرف log میگیریم

$\log\left(\frac{4}{10}\right)^{2x-1} = \log\left(\frac{5}{2}\right)^{3x^2}$

$\Rightarrow 2x-1\left(\log\frac{4}{10}\right) = 3x^2\left(\log\frac{5}{2}\right)$

$2x-1\left(\log 4 - \underbrace{1}_{\log 10}\right) = 3x^2\left(\log 5 - \log 2\right)$

$\Rightarrow 2x-1\left(\cancel{2\log 2 - 1}\right) = 3x^2\left(\cancel{\log\frac{10}{2} - \log 2}\right)$

$-1 \qquad \cancel{1 - 2\log 2}$

$\Rightarrow -2x+1 = 3x^2$

$\Rightarrow 3x^2+2x-1=0 \rightarrow x^2+2x-3=0$

$(x+3)(x-1)=0 \rightarrow x=\frac{1}{3}$ ✓

$\rightarrow x = -\frac{3}{3} = -1$ غ ق ق $\Rightarrow 9x+1>0$

$\log_8 (9\times\frac{1}{3}+1)$

$\Rightarrow \log_8 4 \rightarrow \log_{2^3} 2^2 = \boxed{\frac{2}{3}}$

-۹

$\log_8 b = \frac{2}{3}(1+a) \Rightarrow \log_8 6 = \frac{2}{3}(\log_2 2 + \log_2 3)$

$\Rightarrow \log_8 b = \frac{2}{3}\log_2 6 \Rightarrow \log_8 6 = \log_8 36 \Rightarrow b=36$

$\log(3\underbrace{(36)}_{108} - 8) \Rightarrow \log 100 = \boxed{2}$

-۱۰

$2a = b + c \quad , \quad \frac{2a}{b} = \log 4 \Rightarrow$

$\Longrightarrow 2a = b\log 2$

$\Rightarrow b\log 2 = b + c \Rightarrow c = b\log 2 - b\log 10 \Rightarrow c = b\log 5^{-1}$

$\left(2^{-\frac{1}{8}}\right)^{\frac{c}{a}} \Rightarrow 2^{-\frac{1}{8}\left(\frac{b\log 5^{-1}}{b\log 2^{\frac{1}{2}}}\right)} \Rightarrow 2^{-\frac{1}{8}\left(\log_{2^{\frac{1}{2}}} 5^{-1}\right)}$

$\Rightarrow 2^{-\frac{1}{8}\times -2\left(\log_2 5\right)} \Rightarrow 2^{\frac{1}{4}\left(\log_2 5\right)} \Rightarrow 5^{\frac{1}{4}} = 5^{\frac{1}{4}}$

$= \boxed{\sqrt[4]{5}}$