امید محمدفرجادی

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$f(x)=a^{Ax+B}$ $\quad y=x^2$ $\quad 1,2$

$\to a^{2A+B}=a^2 \to 2A+B=2 \to 1=2-2A \quad \to a^{A+B}=a^0=1 \to A+B=0 \to A+2-2A=0 \to -A=-2$

$A=1$ ✓

$B=-1$ ✓

$f(x)=a^{x+1} \to 0 \to a^{-1} \to \frac{1}{2}$ ✓

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$\log_2^{(2^x+10)}=x+2 \to 2^x+10=2^{x+2} \to 2^x+10=2^x\cdot 2^2 \to 2^x=A$

$A^2+10=8A \to A^2-8A+10=0$

$(A-2)(A-5)=0 \to A=2,5$

$\to 2^x=2 \to \log_2^2=x$

$\to 2^x=5 \to \log_2^5=x$

$\to \log_2^2+\log_2^5 \to \log_2^{10}$ ✓

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$(\log_{21}^3)^2+\log_3\log_{21}^{441}$

$\log_{21}^3=t_1 \quad \log_{21}^7=t_2$

$\log_{21}^{441} \to 2\log_{21}^{21^2}=4$

$\log_{21}^{147} \to \log_{21}^3+\log_{21}^7=t_1+t_2$

$t_1^2+(t_1+t_2)(t_1+t_2)=t_1^2+2t_1t_2+t_2^2+t_1^2+t_1t_2+t_1t_2+t_2^2$

$=2t_1^2+2t_2^2+3t_1t_2=2(t_1^2+t_2^2+t_1t_2)=2(4-t_1t_2)$

$\log_{21}^{21}=1 \to \log_{21}^3+\log_{21}^7=1 \to t_1+t_2=1$

$t_1t_2=0 \to t_1=0, t_2=1$ یا برعکس

$2(4-0)=8$ ✓

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$\log(x^2-3x+1)+2\log(1-x)=0 \quad \log_2^{-x}=?$

$\log(x-1)^2 \to 2\log x-1+2\log 1-x=0 \to 2\log 1-x=0 \to \log 1-x=1 \to 1-x=10 \to x=-9$ ✓

$\log_2^{-(-9)} \to \log_2^9 \to 2$ ✓

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$\log_2^{(x^2+3x+3)}+\log_2^{(x-2)}=2 \quad \log\frac{x}{\sqrt{2}} \to \log\frac{x}{2^{\frac{1}{2}}} \to 2\log_2^x=?$

$\log_2^{(x^2+3x+3)(x-2)}=2 \to \log_2^{x^3-2x^2+3x^2-6x+3x-6}=2 \to \log_2^{x^3-3x-6}=2 \to x^3-3x-6=4 \to x^3=13 \to x=\sqrt[3]{13}$

$\log_2^x \to \log_2^{\sqrt[3]{13}} \to \log_2^{13^{\frac{1}{3}}} \to \frac{1}{3}\log_2^{13}$

$\log_2^{13}=3 \to \frac{1}{3}\cdot 3=1$ ✓

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$\log(x+8)^{-3}\log\frac{1}{(x-2)^2}=2 \quad \log_{\frac{1}{2}}^{-x}=?$

$x-2=8$

$\log^{-3}-\log\frac{1}{8^2}=2 \to \log\frac{-3}{8^2} \to \log(-8)^2=2 \to 100=-A^2 \to -100=A^2 \to A=-10$

$x-2=-10 \to x=-8$ ✓

$\log_{\frac{1}{2}}^{-(-8)} \to \log_{2^{-1}}^{2^3} \to -3\log_2^2 \to -3$ ✓

امیر محمد زرجانی

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$\log_{\sqrt{5}}^{n-2} = ?$ $\quad 3^{n^2-2} = 81^n \to 3^{n^2-2} = 3^{4n} \to n^2-2 = 4n \to n^2-4n-2=0 \to \Delta \to 16-4(1)(-2) \to 24$

$\frac{4 \pm \sqrt{24}}{2} \to 2+\sqrt{6} = n$ ✓ $\quad -2$ ✗

$\log_{\sqrt{5}}^{2+\sqrt{6}} \to \log_{\sqrt{5}}^{n} \to \log_{5^{\frac{1}{2}}}^{5} \to \frac{1}{\frac{1}{2}} \log_5^5 \to \frac{1}{2}$ ✓

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$\log_{27}^{25} = \frac{5}{3}$ $\quad \log_{125}^{8} = ?$

$\to \log_{3^3}^{2^3} \to \frac{3}{3} \log_3^2 \to \frac{3}{5}$ ✓

$3^{\frac{5}{3}} = 2$

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$\log_{\frac{1}{4}}^{3} \to 0.8$ $\quad \log_{16}^{9} = ? \to \log_{4^2}^{3^2} \to \log_{(\frac{1}{4})^2}^{3^2} \to \log_{\frac{1}{4}}^{3} \to \frac{1}{2}$

$\log_{\frac{1}{4}}^{3} = 0.8 \to 3 = (\frac{1}{4})^{0.8}$

$\frac{1}{2} \log_2^3 \to \frac{3}{2}$ ✓

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$(a \log_2 x^2 + ax + b \log_2 = 0 \to -1) \quad (5^{\frac{b}{a}} = ?)$

$\to a \log_2 - a + b \log_2 = 0 \to (a+b) \log_2 - a = 0 \to (a+b) \log_2 = a = a \log_2 + b \log_2$

$\to b \log_2 = a(1 - \log_2) \to \frac{b}{a} \to \frac{1-\log_2}{\log_2} \to 5^{\frac{b}{a}} \to 2^{\frac{b}{a}} \to 2^{\frac{1-\log_2}{2\log_2}}$

$\to 2^{(-1+\frac{1}{\log_2})\frac{1}{2}} \to 2^{(-1+\log_2 10)\frac{1}{2}} \to 2^{\log_2 10} \to 10^{\frac{1}{2}\log_2 1} \to 10^{-1} = \frac{10}{2} = 5$

$5^{\frac{b}{a}} \to (5)$ ✓

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