یازدهم تجربی B

تکلیف

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$f(x)=3^{Ax+B}$ $\quad y=x^2 \quad (1,2)$

$\Rightarrow 3^{2A+B}=3^2 \to 2A+B=2 \to B=2-2A$

$\downarrow \; 3^{A+B}=1 \Rightarrow 3^{A+B}=1 \to A+B=0 \to A+2-2A=0 \to -2A=-2$

$A=1 \checkmark$

$B=-1 \checkmark$

$f(x)=3^{x-1} \to 3^{-1} \to \boxed{\frac{1}{3}}$

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$\log_3(4^x+15)=x+2 \to 4^x+15=2^{x+3} \to 2^{2x}+15=2^{x+3}$ $\quad 2^x=A$

$\to A^2+15=8A \to A^2-8A+15=0$

$(A-3)(A-5)=0$

$\to 2^x=3 \to \log_2 3 = x$

$\to 2^x=5 \to \log_2 5 = x \to \log_2 3+\log_2 5 \to \boxed{\log_2 15}$

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$(\log_{21} 3)^2+\log_2 147 \cdot \log_{21} 1323 \to$ $\quad \log_{21}7=t_1 \quad \log_{21}3=t_2$

$\log_{21} 1323 \to 2\log_{21}7+3\log_{21}3 \to 2t_1+3t_2$ $\quad \log_{21} 147 \to 2\log_{21}7+\log_{21}3 \to 2t_1+t_2$

$t_2^2+(2t_1+3t_2)(2t_1+t_2) \to t_2^2+4t_1^2+8t_1t_2+3t_2^2$ $\quad \log_{21}21=1 \to \log_{21}3+\log_{21}7$

$\to 4t_2^2+8t_1t_2+4t_1^2 \to 4(t_1+t_2)^2 \to \boxed{4}$ $\quad t_1+t_2=1$

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$\log(x^2-2x+1)+3\log(1-x)=5 \quad \log_3^{-x}=?$

$\log(x-1)^2 \to 2\log x-1+3\log 1-x=5 \to 5\log 1-x=5 \to \log 1-x=1 \to 1-x=10 \to x=-9 \checkmark$

$\log_3 -(-9) \to \log_3 3^2 \to \boxed{2}$

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$\log_2(x^2+2x+4)+\log_2(x-2)=3$ $\quad \log_{\sqrt{2}}\frac{x}{\sqrt{2}} \to \log_{2^{\frac{1}{2}}} \to 3\log_2 x=?$

$\log_2(x^2+2x+4)(x-2)=3 \to \log_2 x^3-2x^2+2x^2+4x-4x-8 \to \log_2 x^3-8=3 \to x^3-8=8 \to x^3=16 \to x=\sqrt[3]{16}$

$\log \frac{x}{\sqrt{2}} \to \log \frac{\sqrt[3]{16}}{\sqrt{2}} \to \log_{2^{\frac{1}{2}}} 2^{\frac{4}{3}} \to \frac{4}{3} \to 4\log_2 2 \to \boxed{4}$

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$\log(x-2)^{?} \quad \log \frac{1}{(x-2)^2}=3 \quad \log \frac{-x}{\sqrt{2}}=?$

$x-2=A$

$\log -A-\log\frac{1}{A^2}=3 \to \log \frac{-A}{\frac{1}{A^2}} \to \log -A^3=3 \to 1000=-A^3 \to -1000=A^3 \to A=-10$

$x-2=-10 \to x=-8 \checkmark$ $\quad \log \frac{-(-8)}{\sqrt{2}} \to \log_{2^{\frac{1}{2}}} 2^3 \to \boxed{6\log_2 2} \to \boxed{6}$

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$\log_{6}^{x-2}=?$ $\quad 3^{x^2-2}=81^{x}\to 3^{x^2-2}=3^{4x}\to x^2-2=4x\to x^2-4x-2=0\to \Delta\to 16-4(1)(-2)\to 24$

$\frac{4\pm\sqrt{24}}{2}\to 2+\sqrt{6}=x$ ✓ $\quad 2-\sqrt{6}$ ✗

$\log_{6}^{2+\sqrt{6}-2}\to \log_{6}^{\sqrt{6}}\to \log_{6}^{6^{\frac{1}{2}}}\to \frac{1}{2}\log_{6}^{6}\to \boxed{\frac{1}{2}}$

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$\log_{3}^{2}=\frac{5}{8}$ $\quad \log_{18}^{8}=?$

$\to \log_{3^2\times 2}^{2^3}\to \frac{3^{(\frac{5}{8})3}}{3^2\times 3^{\frac{5}{8}}}\to \frac{3^{\frac{15}{8}}}{3^{\frac{21}{8}}}\to \frac{5}{7}\log_{3}^{3}\to \boxed{\frac{5}{7}}$

$3^{\frac{5}{8}}=2$

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$\log_{4}^{3}\to 0/8$ $\quad \log_{12}^{6}=?\to \log_{3\times2\times2}^{3\times2}\to \log_{2^{1/6}\times2\times2}^{2^{1/6}\times2}\to \log_{2^{3/6}}^{2^{2/6}}\to \frac{26}{36}\to \frac{13}{18}$

$\log_{2}^{3}=0/8\to 3=2^{1/6}$ $\quad \frac{13}{18}\log_{2}^{2}\to \boxed{\frac{13}{18}}$

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$(a\log 2)x^2+ax+b\log 2=0 \quad -1$ $\quad (\sqrt{2})^{\frac{b}{a}}=?$

$\to a\log 2-a+b\log 2=0\to (a+b)\log 2-a=0\to (a+b)\log 2=a=a\log 2+b\log 2$

$\to b\log 2=a(1-\log 2)\to \frac{b}{a}\to \frac{1-\log 2}{\log 2}\to \sqrt{2}^{\frac{b}{a}}\to 2^{\frac{b}{2a}}\to 2^{\frac{1-\log 2}{2\log 2}}$

$\to 2^{(-1+\frac{1}{\log 2})\frac{1}{2}}\to 2^{(-1+\log_2 10)\frac{1}{2}}\to 2^{\log_2 \frac{10}{2}}\to 10^{\log 5}\to 10\to 2^{\frac{1}{\log 2}-1}=\frac{10}{2}=5$

$\sqrt{2}^{\frac{b}{a}}\to \boxed{\sqrt{5}}$