ه) اسم محمدزاده ۲۴ یازده B

۱)

$x=1 \rightarrow f(x)=1 \Rightarrow 3^{A+B}=1 \Rightarrow A+B=0$

$x=3 \rightarrow f(x)=9 \Rightarrow 3^{3A+B}=9 \Rightarrow 3A+B=2$

$\Rightarrow 2A=2 \Rightarrow A=1 \Rightarrow B=-1$

$\Rightarrow x=0 \Rightarrow f(x)=3^{1(0)-1}=\frac{1}{3}$ $\quad (0, \frac{1}{3})$

۲)

$2^{\log_2(4^x+15)}=2^{x+3}$

$\Rightarrow 4^x+15=8\times 2^x$

$\Rightarrow 2^{2x}-8\times 2^x+15 \Rightarrow t=2^x \Rightarrow t^2-8t+15=0$

$\Rightarrow t=\frac{8\pm 2}{2}=5, 3 \Rightarrow 2^x=5, 3 \Rightarrow \boxed{x=\log_2 5, \log_2 3 \quad \alpha+\beta=\log_2 15}$

۳)

$a=(\log_{21}3)^2+\log_{21}147\,\log_{21}1323=(\log_{21}3)^2+\log_{21}147\times\log_{21}(9\times147)$

$=(\log_{21}3)^2+\log_{21}(7\times21)\,\log_{21}(9\times7\times21)=(\log_{21}3)^2+(\log_{21}7+1)(2+\log_{21}3$

$=(\log_{21}3)^2+(2-\log_{21}3)(2+\log_{21}3)=4-(\log_{21}3)^2+(\log_{21}3)^2$

$=4$

۴)

$10^5=(x^2-2x+1)(1-x)^3$

$\Rightarrow (x-1)^2\times(1-x)^3=(1-x)^5=10^5 \Rightarrow 1-x=10$

$\Rightarrow \boxed{x=-9} \Rightarrow \log_3 9=2$

۵)

$a=\log_2(x^2+2x+4)+\log_2(x-2)=3$

$2^a=8=(x^2+2x+4)(x-2)=x^3-8=8$

$\Rightarrow x^3=16 \Rightarrow x=2^{\frac{4}{3}} \Rightarrow \log_{\sqrt[3]{2}} 2^{\frac{4}{3}}=4$

(6)

$$\log(2-x)-\log\frac{1}{(x-2)^2}=3=\log(2-x)+\log(x-2)^2$$

$$=\log(2-x)^3=3 \Rightarrow 10^3=1000=(2-x)^3 \Rightarrow 10=2-x$$

$$\Rightarrow x=-8 \Rightarrow \log_{\sqrt{2}} 8=6$$

(7)

$$3^{x^2-2}=3^{4x} \Rightarrow 4x=x^2-2$$

$$\Rightarrow x^2-4x-2=0 \Rightarrow x=\frac{4\pm 2\sqrt{6}}{2}$$

$$\Rightarrow \log_6 (x-2)=\log_6 \sqrt{6}=\frac{1}{2}$$

(8)

$$3\log_{18} 2=\frac{3}{\log_2 18}=\frac{3}{1+\log_2 9}=\frac{3}{1+2\log_2 3}=\frac{3}{1+\frac{16}{5}}$$

$$=\frac{3}{\frac{21}{5}}=\frac{5}{7}$$

(9)

$$\frac{\log_4 6}{\log_4 12}=\frac{\log_4 2+\log_4 3}{1+\log_4 3}=\frac{\frac{1}{2}+\log_4 3}{1+\log_4 3}=$$

$$=\frac{\frac{1}{2}+0.8}{1+0.8}=\frac{13}{18}$$

15

$$\frac{-a}{a\log 2} = -\log_2 10 = \log_2 \frac{1}{10}$$

$$\frac{\log_2 \frac{1}{10}}{\alpha \circ \beta} - \frac{(-1)}{\alpha} = \beta = \log_2 \frac{1}{5}$$

$$\frac{b\log 2}{a\log 2} = -\log_2 \frac{1}{5}$$

$$\left((\sqrt{2})^{-\log_2 \frac{1}{5}}\right) = 25^{\log_2 \sqrt{2}} = \sqrt{5}$$