B

$f(x)=3^{Ax+B}$, $y=x^2$ $\Longrightarrow$

$3^{Ax+B}=x^2 \xrightarrow{x=1} 3^{A+B}=1 \Rightarrow A+B=0$ (۱)

$3^{Ax+B}=x^2 \xrightarrow{x=3} 3^{3A+B}=3^2 \Rightarrow 3A+B=2$ (۲)

$\ominus$: $2A=2 \Rightarrow \boxed{A=1}$, $\boxed{B=-1}$ ✓

$f(x) \xrightarrow{x=0} 3^{B}=3^{-1}=\frac{1}{3}$ ✓

$\log_2^{(x^2+15)} = x+3 \longrightarrow 2^{(x+3)} = 2^{2x}+15 \xrightarrow{2^x=t} 8t=t^2+15$ (۲)

$\Rightarrow t^2-8t+15 \longrightarrow (t-5)(t-3)=0 \begin{cases} t=5 \\ t=3 \end{cases} \xrightarrow{*} \begin{cases} 2^x=5 \longrightarrow \log_2^5 \\ 2^x=3 \longrightarrow \log_2^3 \end{cases}$

$\log_2^5+\log_2^3=\boxed{\log_2^{15}}$ ✓

$(\log_{21}^3)^2+\log_{21}^{147}\log_{21}^{343} \longrightarrow (\log_{21}^3)^2+\log_{21}^{21\times7}\log_{21}^{21^2\times3}$ (۳)

$\Rightarrow (\log_{21}^3)^2+(1-\log_{21}^3)(2+\log_{21}^3)$

$\longrightarrow (\log_{21}^3)^2+2-(\log_{21}^3)^2 = 2$ (۴) ✓

$(\log_{21}^3+\log_{21}^7)(\log_{21}^{21}+\log_{21}^3)$

$\log_{\frac{21}{3}}^{21} = \log_{21}^{21}-\log_{21}^3$

$\log^{(x^2-2x+1)}+2\log^{(1-x)}=5 \xrightarrow{(x-1)^2=(1-x)^2}$ (۴) $\log^{(1-x)}+2\log^{(1-x)}=5$ (۴)

$\Rightarrow 2\log^{(1-x)}+2\log^{(1-x)}=5 \longrightarrow \log^{(1-x)}=1 \rightsquigarrow 1-x=10$

$\boxed{x=-9}$ ✓

$\log_3^{(-x)} = 2$, $-(-9)=9$

$\Rightarrow$ (۲) ✓

۵) $\log_2^{(x^2+2x+6)} + \log_2^{(x-2)} = 3 \xrightarrow{\text{اتحاد مزدوج}} \log_2^{x^3-8} = 3 \longrightarrow x^3-8 = 8$ (۲)

$x^3 = 16$

$x = 2^{\frac{4}{3}}$ ✓

$\log_{\sqrt{2}}^{x} \longrightarrow \log_{2^{\frac{1}{2}}}^{2^{\frac{4}{3}}} =$ ④ ✓

۶) $\log^{(2-x)} - \log^{\frac{1}{(x-2)^2}} = 3 \xrightarrow{(2-x)^2=(x-2)^2} \log^{(2-x)} - \log^{(2-x)^{(-2)}} = 3$ (۲)

$\log^{(2-x)} = 1 \longrightarrow 2-x = 10$

$x = -8$ ✓

$\log_{\sqrt{2}}^{(-x)} = \log_{2^{\frac{1}{2}}}^{2^3} =$ ④ ✓

۷) $3^{x^2-2} = 81^{x} \longrightarrow 3^{x^2-2} = 3^{4x} \longrightarrow x^2-4x-2=0$ (۲)

$\Delta = 16+8 = 24 \Rightarrow x_{1,2} = \frac{4 \pm \sqrt{24}}{2}$

$\longrightarrow 2 \pm \sqrt{6} \Rightarrow x = 2+\sqrt{6}$ (غ ق ق)

$\log_4^{(x-2)} = \log_4^{\sqrt{6}} =$ ① ✓

$\log_3^2 = \frac{5}{8}$

۸) $\log_{18}^{8} = \frac{\log_3^8}{\log_3^{18}} = \frac{3\log_3^2}{\log_3^9 + \log_3^2} = \frac{\frac{15}{8}}{2+\frac{5}{8}} = \frac{15}{21} = \frac{5}{7}$ (۲) ✓

$\log_6^3 = \frac{8}{10}$

۹) $\log_{12}^{6} = \frac{\log_6^6}{\log_6^{12}} = \frac{\log_6^3 + \log_6^2}{\log_6^3 + 2\log_6^2} = \frac{\frac{8}{10}+\frac{5}{10}}{\frac{8}{10}+\frac{10}{10}} = \frac{13}{18}$ (۲) ✓

۱۰) $(a\log^2)x^2 + ax + b\log^2 = 0 \xrightarrow{x=-1} a\log^2 - a + b\log^2 \longrightarrow \log^2(a+b) = a$ (۲)

$\Rightarrow \log^2 = \frac{a}{a+b} \longrightarrow \log_2^{10} = \frac{a+b}{a}$

$\log_2^{10} = 1 + \frac{b}{a} \longrightarrow \log_2^5 = \frac{b}{a} \longrightarrow 2^{\frac{b}{a}} = 5^*$

$(\sqrt{2})^{\frac{b}{a}} = (2^{\frac{1}{2}})^{\frac{b}{a}} = (2^{\frac{b}{a}})^{\frac{1}{2}} \xrightarrow{*} 5^{\frac{1}{2}} = \sqrt{5}$ ✓