نام و نام خانوادگی ...آرین ...... پاسخنامه تشریحی تکلیف شماره ...۲۴... کلاس ...یازدهم...

۱۹,۵

۱ (۲)

$\xrightarrow{x=1} y=1 \,,\, f(x)=3^{A+B} \Rightarrow 3^{A+B}=1 \Rightarrow A+B=0$

$\xrightarrow{x=3} y=9 \,,\, f(x)=3^{3A+B} \Rightarrow 3^{3A+B}=9 \Rightarrow 3A+B=2$

$2A=2 \Rightarrow A=1 \,,\, B=-1$

$\rightarrow f(x)=3^{x-1}$ $\xrightarrow{\text{عرض از مبداء}}$ $(x=0) \rightarrow f(0)=3^{-1}=\frac{1}{3}$

۲ (۲)

$\log_2 (4^x+15)=x+3 \Rightarrow 2^{x+3}=4^x+15 \rightarrow 2^x\times 8=(2^x)^2+15 \rightarrow 2^x=t$

$\Rightarrow t^2-8t+15=0 \rightarrow (t-3)(t-5)=0 \;<\; \begin{array}{l} t=3 \rightarrow 2^x=3 \rightarrow x=\log_2 3 \\ t=5 \rightarrow 2^x=5 \rightarrow x=\log_2 5 \end{array}$

$\rightarrow x_1+x_2=\log_2 3+\log_2 5=\log_2 15$

۳ (۲)

$\log_{21}3\times\log_{21}3+\underbrace{(1+\log_{21}7)(\cancel{3}+\log_{21}3)}$

$2+\log_{21}3+\underbrace{2\log_{21}7}_{\log_{21}49}+\log_{21}3\,\log_{21}7=2+\log_{21}147+\log_{21}3\times\log_{21}7$

$\log_{21}3\,(\underbrace{\log_{21}3+\log_{21}7}_{\log_{21}21=1})+2+\log_{21}147=\log_{21}3+2+1+\log_{21}7=3+\log_{21}21=4$

۴ (۱,۵)

$\log(x^2-2x+1)=\log(1-x)^2=2\log(1-x)$

$\rightarrow 2\log(1-x)+3\log(1-x)=5 \rightarrow 5\log(1-x)=5 \rightarrow \log_{10}(1-x)=1$

$\rightarrow 1-x=10 \Rightarrow x=-9$ $\quad \log_3^{-x}=\log_3 9=\boxed{2}$

۵ (۲)

$\log_2(x^2+2x+4)+\log_2(x-2)=3 \rightarrow \log_2 (x-2)(x^2+2x+4)=3$

$\log_2(x^3-8)=3 \rightarrow x^3-8=8 \rightarrow x^3=16 \rightarrow x=\sqrt[3]{16}$

$\log_{\sqrt[3]{2}} x=\log_{\sqrt[3]{2}}\sqrt[3]{16}=\log_{2^{\frac13}}16^{\frac13}=\log_2 16=4$

$$\log \frac{1}{(x-2)^2} = \log (x-2)^{-2} = \log (2-x)^{-2} \longrightarrow \log(2-x) - \log (2-x)^{-2} = \log \frac{(2-x)}{(2-x)^{-2}}$$

$$= \log (2-x)^3 = 3 \longrightarrow (2-x)^3 = 10^3 \Longrightarrow 2-x = 10 \rightarrow -x = 8$$

$$\Rightarrow \log_{\sqrt{2}}^{(-x)} = \log_{\sqrt{2}}^{8} = \log_{2^{1/2}}^{2^3} = 6\log_2^2 = \boxed{6}$$

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۶

$$3^{x^2-2} = 3^{4x} \longrightarrow x^2-2 = 4x \longrightarrow x^2-4x-2=0 \longrightarrow x = \frac{4\pm\sqrt{24}}{2} = 2\pm\sqrt{6}$$

$$\log_6^{(x-2)} \longrightarrow x-2>0 \longrightarrow x>2 \longrightarrow \begin{cases} x = 2-\sqrt{6} \; \times \\ x = 2+\sqrt{6} \; \checkmark \end{cases} \Longrightarrow x-2 = 2+\sqrt{6}-2 = \sqrt{6}$$

$$\longrightarrow \log_6^{(x-2)} = \log_6^{\sqrt{6}} = \boxed{\frac{1}{2}}$$

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$$\log_{18}^{8} = \frac{\log_3^8}{\log_3^{18}} = \frac{3\log_3^2}{\log_3^9+\log_3^2} = \frac{3\times\frac{5}{8}}{2+\frac{5}{8}} = \frac{\frac{15}{8}}{\frac{21}{8}} = \frac{15}{21} = \boxed{\frac{5}{7}}$$

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$$\log_{12}^{6} = \frac{\log_4^6}{\log_4^{12}} = \frac{\log_4^2+\log_4^3}{\log_4^3+\log_4^4} = \frac{\frac{1}{2}+\frac{8}{10}}{\frac{8}{10}+1} = \frac{1/3}{1/8} = \boxed{\frac{13}{18}}$$

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۹

$$\xrightarrow{x=-1} a\log 2 - a + b\log 2 = 0 \rightarrow \log 2^a + \log 2^b - a = 0 \rightarrow \log_{10}^{2^{a+b}} = a$$

$$\rightarrow 2^{a+b} = 10^a \longrightarrow 2^a \times 2^b = 2^a \times 5^a \Rightarrow 2^b = 5^a$$

$$\rightarrow (\sqrt{2})^{b/a} = (2^{1/2})^{b/a} = 2^{\frac{b}{2a}} = (2^b)^{\frac{1}{2a}} = (5^a)^{\frac{1}{2a}} = 5^{\frac{a}{2a}} = 5^{\frac{1}{2}} = \boxed{\sqrt{5}}$$

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