۲۴ / /   ۲۰   سمیرعلای

۱)

$f(x) = 3^{Ax+B}$

$y = x^2$

(۲)

$3^{Ax+B} = x^2$ if $x=3$, $x=1$

$3^{A+B} = 1 \Rightarrow A+B=0$
$3^{3A+B} = 9 \Rightarrow 3A+B=2$
$\Rightarrow 2A=2$, $A=1 \Rightarrow B=-1$

$f(x) = 3^{x-1} \xrightarrow{x=0} 3^{-1} = \boxed{\frac{1}{3}}$ ✓

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۲)

(۲)

$\log_2^{(4^x+15)} = x+3$

$2^{x+3} = 4^x + 15$

$2^{x+3} = 2^{2x} + 15 \xrightarrow{\div 2^{2x}} 2^{3-x} = 1 + \frac{15}{2^{2x}}$

$8 \times 2^x = (2^x)^2 + 15$

$8t = t^2 + 15$

$t^2 - 8t + 15 = 0 \Rightarrow t=5$, $t=3$

$2^x = 3$, $\log_2^3 = x$
$2^x = 5$, $\log_2^5 = x$
$t \Rightarrow \boxed{\log_2^{15}}$ ✓

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۳)

(۲)

$(\log_{21}^3)^2 + \log_{21}^{147}(\log_{21}^{147} + \log_{21}^3 + \log_{21}^3)$

$\log_{21}^{147} = \log_{21}^{21} + \log_{21}^7 = 1 + \log_{21}^7 = 1 + \log_{21}^{21} - \log_{21}^3 = 2 - \log_{21}^3$

$\log_{21}^{1323} = \log_{21}^{147} + \log_{21}^9 = 2 - \log_{21}^3 + \log_{21}^3 + \log_{21}^3 = 2 + \log_{21}^3$

مزدوج

$(\log_{21}^3)^2 + 4 - (\log_{21}^3)^2 = (4)$ ✓

$\boxed{۴}$

$\log(x^2-7x+1) + \log(1-x)^2 = 0 \Rightarrow$ ۴

$\log(1-x)^2$

$\log(1-x)^0 = 0$ $\quad \log(1-x)^0 = \log 1^{000}$

$(1-x)^0 = 1^0 \Rightarrow 1-x = 1$

$x = -9$ ✓ $\quad \log_{\sqrt{2}}^{-(-9)} = ۴$ ✓ (۲)

$\log_{\sqrt{2}}^{(x^2+2x+3)(x-2)} = 2^2 \Rightarrow \sqrt{2}^2 = 0$ ۵

$(x^2+2x+3)(x-2) = 1 \Rightarrow x^3+2x^2+3x-2x^2-3x-1 = 0$

$x^3-1 \Rightarrow x^3-1=1$

$x^3 = 16$

$x = \sqrt[3]{16}$ $\quad \log_{\sqrt{2}}^{\sqrt[3]{16}} = ۳$ ✓ (۳)

۶

$\log(2-x) + \log(x-2)^2 = 2^2$

$\log(2-x)^2 = \log 1^{000} \Rightarrow 2-x = 1^0$

$-1 = x$ ✓

$\log_{\sqrt{2}}^{(-x)} = \log_{\sqrt{2}}^{1} = ۰$ ✓ (۴)

$\log_5 x-2 = ?$

$3^{x^2-x} = 3^{3x} \Rightarrow$ ۸

$x^2-x = 3x$

$x^2-4x = 0$

$x = \frac{4 \pm \sqrt{16}}{2} \Rightarrow$ (۲)

$x = 2 \pm \sqrt{5} \Rightarrow \log_5^{(x-2)} = \begin{cases} \log_5^{2+\sqrt{5}} \Rightarrow \frac{1}{2} \checkmark \\ \log_5^{2-\sqrt{5}} \Rightarrow X \end{cases}$

۷- ✗

$3^{\frac{5}{7}} = 2 \Rightarrow 8 = 3^{\frac{21}{5}}$

$\log_{11}^8 = \log_{9\times2}^8 = \log_{9\times2}^{3^{\frac{21}{5}}} = \frac{21}{5}\log_{11}^3$ (۲)

$\log_3^2 \cdot \log_2^4 = \log_3^4 = \frac{21}{5} = \log_3^8 = \frac{21}{7}$

$\log_3^{11} = \log_3^2 + \log_3^4 + \log_3^5 = 2 + 2\log_3^2 = 2 + \frac{5}{4} = \frac{13}{4}$

$\Rightarrow \frac{\log_3^8}{\log_3^{11}} = \frac{\frac{15}{7}}{\frac{13}{4}} = \frac{15}{13}$ ✓ (۲)

۹-

$3^{\frac{1}{10}} = 2 \Rightarrow 3 = 2^{\frac{10}{7}}$

$\log_{12}^6 = \log_{2^2\times2^{\frac{1}{7}}\times2}^{2^{\frac{10}{7}}} = \log_{2^{\frac{24}{7}}}^{2^{\frac{10}{7}}} = \frac{10}{24} \times \log_2^2 = \frac{10}{24} = \frac{5}{12}$ ✓ (۲)

$(a\log 2)x^2 + ax + b\log 2 = 0 \xrightarrow{x=-1} \Rightarrow$ -1.

$a\log 2 + b\log 2 - a = 0 \xrightarrow{\div a} \Rightarrow$ (۲)

$\log 2 + \frac{b}{a}\log 2 - 1 = 0$

$\log 2 + \frac{b}{a}\log 2^{-1} = \log 1 .$

$\log 2 + \frac{b}{a}\log 2 - 1 = 1 - (\log 2 + \log a)$

$\log 2 + \frac{b}{a}\log 2 - 1 = \log 2 + \log a - 1$

$\frac{b}{a} = \frac{\log a}{\log 2} = \log_2 a$ ✓

$\sqrt{2}^{\log_2 a} = a^{\log_2 \sqrt{2}} =$

$\sqrt{a}$ ✓