





$$(\lg_{r_1}^r)^r + \lg_{r_1}^{r \times r_1} \lg_{r_1}^{r_1 \times r_1} = (\lg_{r_1}^r)^r + (\lg_{r_1}^{\frac{r}{r}} + 1)(\lg_{r_1}^{r_1 \times r} + 1)$$

$$(\lg_{r_1}^r)^r + (1 - \lg_{r_1}^r + 1)(1 + \lg_{r_1}^r + 1) =$$

$$(\lg_{r_1}^r)^r + (r - \lg_{r_1}^r)(r + \lg_{r_1}^r) = (\lg_{r_1}^r)^r + r - (\lg_{r_1}^r)^r = \boxed{r}$$