محمد امین جعفری — تکلیف ۲ — یازدهم ر B

۱

$x=1 \longrightarrow y=1 \Rightarrow 3^{Ax+B} = 3^{A+B} = 1 \Rightarrow A+B=0$

$x=2 \longrightarrow y=9 \Rightarrow 3^{Ax+B} = 3^{2A+B} = 3^2 \Rightarrow 2A+B=2$ $\Big\} \Rightarrow A=1 \Rightarrow B=-1$

$\Rightarrow f(x)=3^{x-1} \xrightarrow{x=0} f(x)=3^{-1}=\frac{1}{3}$

۲

$\Rightarrow 2^{(x+3)} = 4^x + 15 = 2^{2x} + 15 \Rightarrow 2^{(x+3)} - 2^{2x} = 15 \Rightarrow 2^x(2^3 - 2^x) = 15$

$\xrightarrow{2^x = t} 8t - t^2 = 15 \Rightarrow t^2 - 8t + 15 = 0 \Rightarrow t = \begin{cases} 3 \\ 5 \end{cases} \Rightarrow x = \log_2 t$

$x_1 + x_2 = \log_2 3 + \log_2 5 = \log_2 (3\times 5) = \log_2 15$

۳

$= \log_{21} 3 \times \log_{21} 4 + \log_{21} (21\times 7) \times \log_{21} (21\times 21\times 3) = (\log_{21} 3)^2 + (1+\log_{21} 7)(1+1+\log_{21} 3)$

$\xrightarrow[\log_{21} 7 = k]{\log_{21} 3 = t} t^2 + (k+1)(t+2) = t^2 + (k+1)t + 2(k+1)$

$\Rightarrow t = \frac{-k-1 \pm \sqrt{(k+1)^2 - 4(1)(2k+2)}}{2} = \frac{-k-1 \pm \sqrt{k^2-6k-7}}{2} \Rightarrow 2t+k+1 = \sqrt{k^2-6k-7}$

$\Rightarrow k^2-6k-7 = (2t+k+1)^2 = 4t^2 + k^2 + 2k + 1 + 4kt + 4t \Rightarrow$

۴

$= \log (x-1)^2 + \log (1-x)^3 = \log (1-x)^2(1-x)^3 = \log (1-x)^5 = 5 \Rightarrow 1-x = 10 \Rightarrow x = -9$

$\Rightarrow \log_3 (-x) = \log_3 9 = 2$

۵

$\log_2 (x^2+2x+4)(x-2) = 4 \Rightarrow x^3 - 2x^2 + 2x^2 - 4x + 4x - 8 = x^3 - 8 = 8 \Rightarrow x^3 = 16 = 2^4$

$\Rightarrow x = 2^{\frac{4}{3}} \Rightarrow \log_{\sqrt{2}} x = \log_{2^{\frac{1}{2}}} 2^{\frac{4}{3}} = \frac{\frac{4}{3}}{\frac{1}{2}} \times \log_2 2 = \frac{8}{3}$

۶ $\log_{(x-2)^2}^{(2-x)} = \log_{\frac{1}{(x-2)^2}}^{\ } = \log^{\frac{2-x}{\frac{1}{(x-2)^2}}} = \log\left((2-x)(x-2)^2\right) = \log\left((2-x)(2-x)^2\right) = \log(2-x)^3 = 3$

$\Rightarrow 2-x = 10 \Rightarrow x = -8 \Rightarrow \log_{\sqrt{2}}^{(-x)} = \log_{\sqrt{2}}^{8} = 6$

۷ $3^{x^2-2} = (3^4)^x = 3^{4x} \Rightarrow x^2-2 = 4x \Rightarrow x^2-4x-2=0 \Rightarrow x = \frac{4 \pm \sqrt{16-4(-2)(1)}}{2}$

$= 2 \pm \frac{\sqrt{12}}{2} = 2 \pm \sqrt{3} \xrightarrow[x>2]{x-2>0} x = 2+\sqrt{3} \Rightarrow \log_6^{(x-2)} = \log_6^{\sqrt{3}}$

۸ $\frac{1}{\log_3^2} = \log_2^3 = \frac{8}{5}$

۹ $\Rightarrow \frac{1}{\log_4^3} = \log_3^4 = \frac{10}{8} \Rightarrow \log_3^{12} = \log_3^{4\times3} = \log_3^4 + \log_3^3 = \frac{10}{8} + 1 = \frac{18}{8}$

۱۰ $\Rightarrow a\log 2 + b\log 2 = a \Rightarrow (a+b)\log 2 = a \Rightarrow \log 2 = \frac{a}{a+b} \Rightarrow \log_2^{10} = \frac{a+b}{a}$

$= 1+\frac{b}{a} \Rightarrow \frac{b}{a} = \log_2^{10} - 1 = \log_2^{10} - \log_2^2 = \log_2^{\frac{10}{2}} = \log_2^5$

$\Rightarrow (\sqrt{2})^{\frac{b}{a}} = (\sqrt{2})^{(\log_2^5)} = 5^{(\log_2^{\sqrt{2}})} = 5^{\frac{1}{2}} = \sqrt{5}$