

19, 2

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① $x=0 \rightarrow y = 2^x - \log^{-b} c = 2 \rightarrow \log^{-b} c = -1 \rightarrow +b = -\frac{1}{c}$ (1, VA)

$b + c \cdot 2^{\frac{1}{c}} \rightarrow -\frac{1}{c} + c \cdot 2^{-\frac{1}{c}} \rightarrow c + \frac{1}{c} c - 1 = 0 \rightarrow 2c^{\frac{1}{c}} + c - 1 = 0$

$\rightarrow \begin{cases} c = \frac{1}{c} \\ c = -\frac{1}{c} \end{cases} \rightarrow c = \frac{1}{c} \rightarrow b = -1 \checkmark \quad x = -1 \rightarrow y = 2^{-1} - \log^{-1, 8} a - b = 0$

$\rightarrow -10a - b > c \rightarrow -10a + 2 \cdot \frac{1}{c} \rightarrow a = 2 \rightarrow (a+b)b = 2 \cdot 3 = 6$

$\frac{1}{c}(-1) = -3$

$x=0 \rightarrow f(0) = 1 + c x^{\frac{1}{c}} = 2 \rightarrow c x^{\frac{1}{c}} = 1$

$x=1 \rightarrow f(1) = 1 + c x^{\frac{1}{c}} + b = 0 \rightarrow c x^{\frac{1}{c}} + b = -1 \rightarrow b = -1 \checkmark$

$\rightarrow f(-1) = 1 + c x^{\frac{1}{c}} = 1 + \frac{c x^{\frac{1}{c}}}{\frac{1}{c}} = 1 + \frac{1}{\frac{1}{c}} = 1 + c = 1 + \frac{1}{a} = \frac{1}{a}$ (2)

② $x=0 \rightarrow y = c + \log_b \delta = 2 \rightarrow -\log_b \delta + 2 = c$

$x=1 \rightarrow y = c + \log_b^{\frac{1}{c} \delta + b} \delta = 0 \rightarrow 2 - \log_b \delta + \log_b^{\frac{1}{c} \delta + b} \delta = 0$ (1, VA)

$\rightarrow \log_b^{\frac{1}{c} \delta + b} \delta = \log_b \delta - 2 \rightarrow \log_b^{\frac{1}{c} \delta + b} \delta = \log_b \frac{\delta}{\delta^2} \rightarrow \frac{1}{c} \delta + b = \frac{b}{\delta}$

$\frac{a}{b} = -\frac{1}{2} \rightarrow a = -\frac{1}{2} b \rightarrow \frac{a}{b} = -\frac{1}{2}$

③ $f(x) = \log_{\frac{1}{2}} (|x^2 - 1| - x) \rightarrow |x^2 - 1| - x > 0 \rightarrow |x^2 - 1| > x$

$\rightarrow x^2 - 1 > x \rightarrow x^2 - x - 1 > 0 \rightarrow (x+1)(x-2) > 0 \rightarrow \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2}$ (2)

$\rightarrow x^2 - 1 < x \rightarrow x^2 - x - 1 < 0 \rightarrow (x-1)(x+2) < 0 \rightarrow \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$

$\rightarrow D_f = (-\frac{1+\sqrt{5}}{2}, 1) \cup (2, \frac{1+\sqrt{5}}{2}) \checkmark$

④ $x > 1 \rightarrow g(1) = -1 - 3 + 1 = -3 \rightarrow f(1) = 2 + 1^{b-a} \rightarrow 2 > -3 \rightarrow b-a > 1$

$f^{-1}(1) = 1 \rightarrow f(1) = 1 \rightarrow 2 + 1^{b+a} = 1 \rightarrow 1^{b+a} = -1 \rightarrow b+a > 0$

$\rightarrow b-a > 1 \rightarrow b-a \geq 1 \rightarrow a = 1 \checkmark$

$x=1 \rightarrow 0 = -r + (\frac{1}{r})^A + B \rightarrow (\frac{1}{r})^{A+B} = r \rightarrow A+B = -1$ (2)
 $x=r \rightarrow r = -r + (\frac{1}{r})^A + B \rightarrow (\frac{1}{r})^{A+B} = 2r \rightarrow rA+B = -r$
 $A = -1 \rightarrow B = 0$
 $\rightarrow f(x) = -r + (\frac{1}{r})^{-x} \rightarrow f(1) = -r + 1 = 0$ ✓

$(\frac{1}{9})^{\frac{t}{40}} m = \frac{1}{4} m \rightarrow \frac{t}{40} \log \frac{1}{9} = \log \frac{1}{4} \rightarrow \frac{t}{40} \log \frac{1}{9} = \log \frac{1}{4} \rightarrow \frac{t}{40} = \frac{\log \frac{1}{4}}{\log \frac{1}{9}} = \frac{\log 4^{-1}}{\log 3^{-2}} = \frac{-\log 4}{-2 \log 3} = \frac{\log 4}{2 \log 3} = \frac{2 \log 2}{2 \log 3} = \frac{\log 2}{\log 3} = \frac{1}{\log 3} \approx \frac{1}{0.477} \approx 2.1$ (2)
 $\rightarrow \frac{t}{40} \log \frac{1}{9} = \log \frac{1}{4} \rightarrow \frac{t}{40} = \frac{\log 4}{2 \log 3} = \frac{0.602}{2 \cdot 0.477} = \frac{0.602}{0.954} \approx 0.63$
 $t = 0.63 \cdot 40 \approx 25.2$ (19) ✓
 $\rightarrow t = 19$ min ✓

$m(\frac{v}{1})^{\frac{t}{10}} = \frac{1}{2} m \rightarrow (\frac{v}{1})^{\frac{t}{10}} = \frac{1}{2} \rightarrow \frac{t}{10} \log v = \log \frac{1}{2} = -\log 2$ (2)
 $\rightarrow \frac{t}{10} = \frac{-\log 2}{\log v} = \frac{-\log 2}{\log 2} = -1 \rightarrow t = -10$ (1)
 $\rightarrow t = 10$ ✓

$(\frac{45}{100})^t = \frac{1}{2} \rightarrow \frac{t}{100} \log \frac{45}{100} = \log \frac{1}{2} \rightarrow t \log \frac{45}{100} = \log \frac{1}{2} \rightarrow t = \frac{\log \frac{1}{2}}{\log \frac{45}{100}} = \frac{-\log 2}{\log 45 - 2 \log 10} = \frac{-0.301}{0.653 - 2} = \frac{-0.301}{-1.347} \approx 0.22$ (2)
 $\rightarrow t(0.683 - 2) = -\log 2 \rightarrow t(1.317) = 0.301 \rightarrow t = \frac{0.301}{1.317} \approx 0.23$ (15.1)
 $\rightarrow t = 23$ Day ✓

10) a) $y = \log_a x \rightarrow x = \log_a y \rightarrow \frac{dy}{dx} = \frac{1}{x \ln a}$ (2)
 b) $y = \log_a x^2 \rightarrow x^2 = a^y \rightarrow x \neq 0$
 $\log_a x^2 = y \log_a a \rightarrow D_y = \mathbb{R} - \{0\}$
