امیر محمد فروغانی

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۱) $S = \frac{1}{2} \times 4 \times 4 \sin 150 = 8 \times \frac{1}{2} = 4$ ، $4 \times 4 \times \sin\theta = 4 \Rightarrow \sin\theta = \frac{1}{4}$ ، $\cos\theta = \frac{\sqrt{15}}{4}$

$AC^2 = 4^2 + 4^2 - 2 \times 4 \times 4 \times \frac{\sqrt{15}}{4} = 32 - 8\sqrt{15}$ ✓

۲) $S_{ADE} = \frac{1}{2} \sin A \times 4 \times 4 = 8 \sin A$ ، $S_{ABC} = \frac{1}{2} \sin A \times 5 \times 8 = 20 \sin A$

$20\sin A - 8\sin A = 12 \Rightarrow 12 \sin A = 12 \Rightarrow \sin A = 1 \Rightarrow A = 90$ ✓

۳) $\frac{1}{\sqrt{\cos^2 a}} - \tan a = \frac{1}{|\cos a|} - \frac{\sin a}{\cos a} = \frac{1 + \sin a}{|\cos a|}$

$\frac{1}{|\cos a|} - \frac{\sin a}{\cos a} = \frac{1 + \sin a}{|\cos a|} \Rightarrow \cos a < 0$ ، $\frac{|\sin a|}{\cos a} = -\frac{\sin a}{\cos a} \Rightarrow \sin a < 0$

$\cos a < 0$ ، $\sin a < 0$ ⇒ ربع سوم ✓

۴) $\tan\left(\frac{\pi}{2} - a\right) = \cot a$ ، $\cot\theta = \frac{3}{\sqrt{15}} = \frac{\sqrt{3}}{\sqrt{5}}$ ⇒ $\cot a = -\frac{\sqrt{3}}{\sqrt{5}}$ ✓

۵) $\frac{2\cos(24\pi) - 2\sin(15\pi)}{\sin(6\pi) - \cos(19\pi)} = \frac{2\cos\left(\frac{3\pi}{2} - 2\pi\right) - 2\sin(\pi - 2\pi)}{\sin(\pi + 2\pi) - \cos\left(\frac{3\pi}{2} + 2\pi\right)} = \frac{-3\sin 2\pi - 2\sin 2\pi}{-\sin 2\pi - \sin 2\pi} = \frac{5}{2}$ ✓

۶) $\cos a = \frac{2}{3}$ ، $\frac{\sin\left(\frac{\pi}{2} + a\right) - \sin(a - \pi)}{|1 + \tan^2 a - 1|} = \cos a$

$\sin^2\alpha + \cos^2\alpha = 1$ ، $\frac{4}{9}$ ، $\sin^2\alpha = \frac{5}{9} \Rightarrow \sin\alpha = \frac{\sqrt{5}}{3}$

$\Rightarrow \frac{\cos a + \sin a}{\frac{1}{4}} = 2 - \frac{\sqrt{5}}{3} \times 4 \Rightarrow \frac{8 - 4\sqrt{5}}{3}$ ✓

۷) $\sin a = 2\cos a$

$4\cos^2 a + \cos^2 a = 1 \Rightarrow 5\cos^2 a = 1 \Rightarrow \cos^2 a = \frac{1}{5}$ ، $\cos a = \begin{cases} +\frac{1}{\sqrt{5}} \Rightarrow +\frac{\sqrt{5}}{5} \\ -\frac{1}{\sqrt{5}} \Rightarrow -\frac{\sqrt{5}}{5} \end{cases}$

$-\frac{\sqrt{5}}{5}$ ✓

۸) $2mx + (m^2 - 1)y = 2$

$2mx + (m^2 - 1)y - 2 = 0$

$\frac{-a}{b} = \frac{-2m}{m^2 - 1} = \sqrt{2} \Rightarrow 2m = \sqrt{2}m^2 - \sqrt{2} \Rightarrow \sqrt{2}m^2 + 2m - \sqrt{2} = 0$ ، $\Delta = 4 - 4(\sqrt{2})(-\sqrt{2}) = 12$

$\frac{-2 + \sqrt{12}}{2\sqrt{2}} \Rightarrow \frac{2}{2\sqrt{2}} \Rightarrow \frac{1}{\sqrt{2}}$ و $\frac{-6}{2\sqrt{2}} \Rightarrow \frac{-3}{\sqrt{2}}$ ، $\tan\theta = \frac{\sqrt{2}}{2} \times 4 = 2\sqrt{2}$

$\frac{-\frac{3}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = -3$ ✓

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$\tan(\frac{\pi}{4}-\alpha)$ → $\tan\alpha$ / $-\tan\alpha$ (۲)

$\tan(\frac{\pi}{4}-\alpha)=\frac{1-m}{1+m}$

$-\frac{\pi}{4}<\alpha<\frac{\pi}{4}$ ، $\tan=0$

$\frac{1-m}{1+m}>0$ → $\frac{-1 \quad 1}{-1+1-}$ → $(-1,1)$ ✓

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$\tan(300)\cos(210)+\tan(135)\sin(150)$ (۲)

$\frac{480^\circ}{360^\circ}\times\frac{\pi}{1}$ / $\frac{150^\circ}{1}\times\frac{\pi}{180^\circ}$ / $\frac{150^\circ}{2}\times\frac{\pi}{180^\circ}$

$-\sqrt{3}\times-\frac{\sqrt{3}}{2}+-\sqrt{3}\times\frac{\sqrt{3}}{2}$

$-\sqrt{3}(-\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2})=0$ ✓