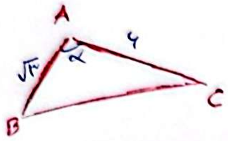
نام و نام خانوادگی ......سهیل بهرامی...... پاسخنامه تشریحی تکلیف شماره ..۲۷.. کلاس .یازدهم [illegible] B.........

۲۰

۱ ۲

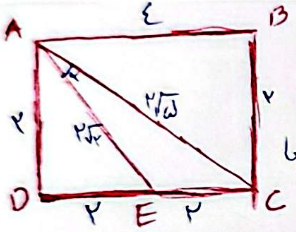


$$S=\frac{۱}{۲}AB\times AC\times\sin\hat{A}=\frac{۱}{۲}\times\sqrt{۳}\times۶\times\sin\hat{\alpha}=۴٫۵$$

$$۳\sqrt{۳}\sin\hat{\alpha}=۴٫۵\Rightarrow\sin\hat{\alpha}=\frac{\sqrt{۳}}{۲}\begin{cases}\to\alpha=۱۲۰^\circ\\\to\alpha=۶۰^\circ\end{cases}$$

$$\frac{۱۲۰}{۶۰}=\textcircled{۲}\ ✓$$

۲ ۲



$$AE^۲=AD^۲+DE^۲=۴+۴=۸\Rightarrow AE=\sqrt{۸}=۲\sqrt{۲}$$

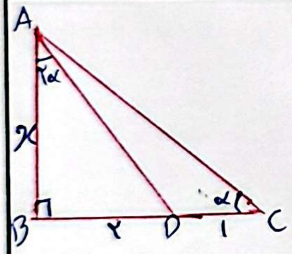
$$AC^۲=AB^۲+BC^۲=۱۶+۴=۲۰\Rightarrow AC=\sqrt{۲۰}=۲\sqrt{۵}$$

$$\xrightarrow[\text{در } E\hat{A}C]{\text{قضیه cosها}}EC^۲=AE^۲+AC^۲-۲AE\times AC\times\cos\alpha\Rightarrow۴=۸+۲۰-۲\times۲\sqrt{۲}\times۲\sqrt{۵}\times\cos\alpha$$

$$\Rightarrow۸\sqrt{۱۰}\cos\alpha=۲۴\Rightarrow\cos\alpha=\frac{۳}{\sqrt{۱۰}}\Rightarrow\sin\alpha=\frac{۱}{\sqrt{۱۰}}\qquad\sin^۲\alpha+\cos^۲\alpha=۱$$

$$\cot\alpha=\frac{\cos\alpha}{\sin\alpha}=\frac{\frac{۳}{\sqrt{۱۰}}}{\frac{۱}{\sqrt{۱۰}}}=\frac{۳}{۱}=\textcircled{۳}\ ✓$$

۳ ۲



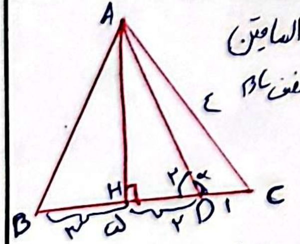
$$\xrightarrow[A\hat{B}C]{\hat{B}=۹۰^\circ}\cot\alpha=\frac{BC}{AB}=\frac{۳}{\underline{x}}\Rightarrow\tan\alpha=\frac{۱}{\cot\alpha}=\frac{x}{\underline{۳}}$$

$$\xrightarrow[A\hat{D}B]{\hat{B}=۹۰^\circ}\cot۲\alpha=\frac{AB}{BD}=\frac{x}{۲}\Rightarrow\tan۲\alpha=\frac{۱}{\cot۲\alpha}=\frac{۲}{\underline{x}}$$

$$\tan۲\alpha=\frac{۲\tan\alpha}{۱-\tan^۲\alpha}=\frac{\frac{۲x}{۳}}{۱-\frac{x^۲}{۹}}=\frac{\frac{۲}{\cancel{۳}}x}{\frac{۹-x^۲}{۳x}}=\frac{۲}{x}\Rightarrow\frac{\overset{۳}{\cancel{۹}}x}{۹-x^۲}=\frac{۲}{x}$$

$$\Rightarrow۳x^۲=۹-x^۲\Rightarrow۴x^۲=۹\Rightarrow x^۲=\frac{۹}{۴}\Rightarrow x=\frac{۳}{۲}=\qquad\cot\alpha=\frac{۳}{x}=\frac{۳}{۱٫۵}=\textcircled{۲}\ ✓$$

۴ ۲



AH در متساوی الساقین ارتفاع، میانه و عمود منصف BC $\Rightarrow BH=CH$

$$\left.\begin{array}{l}BC=BD+DC=۶\\BC=BH+CH=۲CH\end{array}\right\}\Rightarrow۲CH=۶\Rightarrow CH=۳$$

$$CD+DH=۳\Rightarrow DH=۲$$

$$AB^۲=AH^۲+BH^۲\Rightarrow۱۶=۹+AH^۲\Rightarrow AH^۲=۷\Rightarrow AH=\sqrt{۷}$$

$$\tan\hat{D}_۲=\frac{AH}{HD}=\frac{\sqrt{۷}}{۲}\Rightarrow\tan\alpha=\boxed{-\frac{\sqrt{۷}}{۲}}\ ✓$$

۵ ۲

$$۲\sin^۲x+\cos^۲x=\frac{۴}{۳}\qquad\sin^۲x+\cos^۲x=۱\Rightarrow\sin^۲x=\frac{۱}{۳}\qquad\cos^۲x=\frac{۲}{۳}$$

$$\tan^۲x=\frac{\sin^۲x}{\cos^۲x}=\frac{\frac{۱}{۳}}{\frac{۲}{۳}}=\textcircled{\frac{۱}{۲}}\ ✓$$

۶ (۲)

$$\frac{\sin^{۴}\alpha+۴\cos^{۲}\alpha}{۱+\cos^{۲}\alpha}=\frac{(\sin^{۲}\alpha)^{۲}+۴\cos^{۲}\alpha}{۱+\cos^{۲}\alpha}=\frac{(۱-\cos^{۲}\alpha)^{۲}+۴\cos^{۲}\alpha}{۱+\cos^{۲}\alpha}=\frac{\cos^{۴}\alpha-۲\cos^{۲}\alpha+۱+۴\cos^{۲}\alpha}{۱+\cos^{۲}\alpha}$$

$$=\frac{\cos^{۴}\alpha+۲\cos^{۲}\alpha+۱}{۱+\cos^{۲}\alpha}=\frac{(\cos^{۲}\alpha+۱)^{۲}}{\cos^{۲}\alpha+۱}=\underline{\cos^{۲}\alpha+۱}\Bigg\}\quad\frac{\cos^{۴}\alpha+۴\sin^{۲}\alpha}{۱+\sin^{۲}\alpha}=\frac{(۱-\sin^{۲}\alpha)^{۲}+۴\sin^{۲}\alpha}{۱+\sin^{۲}\alpha}$$

$$=\frac{\sin^{۴}\alpha-۲\sin^{۲}\alpha+۱+۴\sin^{۲}\alpha}{۱+\sin^{۲}\alpha}=\frac{\sin^{۴}\alpha+۲\sin^{۲}\alpha+۱}{۱+\sin^{۲}\alpha}=\frac{(۱+\sin^{۲}\alpha)^{۲}}{۱+\sin^{۲}\alpha}=\underline{\sin^{۲}\alpha+۱}$$

$$\cos^{۲}\alpha+۱-\sin^{۲}\alpha-۱=\cos^{۲}\alpha-\sin^{۲}\alpha$$

$$\cos^{۲}\alpha-\sin^{۲}\alpha=\boxed{\cos ۲\alpha}\ ✓$$

۷ (۲)

$$\tan\alpha=\frac{۴}{۳}=\frac{\sin\alpha}{\cos\alpha}\qquad\cot\alpha=\frac{۱}{\tan\alpha}=\frac{۳}{۴}\qquad\sin\alpha\cos\alpha=\frac{۱}{\tan\alpha+\cot\alpha}=\frac{۱}{\frac{۳}{۴}+\frac{۴}{۳}}=\frac{۱۲}{۲۵}$$

$$\underbrace{\sin(\frac{۹\pi}{۲}+\alpha)}_{\cos\alpha}\underbrace{\cos(\frac{۷\pi}{۲}-\alpha)}_{-\sin\alpha}-\underbrace{\tan(\alpha-\frac{۳\pi}{۲})}_{-\cot\alpha}=-\sin\alpha\cos\alpha+\cot\alpha=\frac{-۱۲}{۲۵}+\frac{۳}{۴}=\boxed{\frac{۲۷}{۱۰۰}}\ ✓$$

۸ (۲)

$$۳\cos ۴x+\underbrace{\sqrt{۲}(\sin x-\cos x)}_{۲\sin(x-\frac{\pi}{۴})}=۳\cos ۴x+۲\sin(x-\frac{\pi}{۴})\overset{x=\frac{\pi}{۱۲}}{=\!=}۳\cos\frac{\pi}{۳}+۲\sin(\frac{-\pi}{۶})$$

$$=۳\cos\frac{\pi}{۳}+۲\sin(-\frac{\pi}{۶})=۳(\frac{۱}{۲})+۲(-\frac{۱}{۲})=۱٫۵-۱=\boxed{۰٫۵=\frac{۱}{۲}}\ ✓$$

۹ (۲)

$$\frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha}=\frac{\frac{۸}{۱۵}-\frac{۸}{۱۷}}{\frac{۸}{۱۷}-\frac{۱۵}{۱۷}}=\frac{\frac{۸\times۱۷-۸\times۱۵}{۱۵\times\cancel{۱۷}}}{\frac{-۷}{\cancel{۱۷}}}=\frac{۱۶}{۱۵\times-۷}$$

$$\frac{۱۶}{۱۵\times-۷}=\boxed{\frac{-۱۶}{۱۰۵}}\ ✓$$

$$\tan\alpha=\frac{۲\tan\frac{\alpha}{۲}}{۱-\tan^{۲}\frac{\alpha}{۲}}=\frac{۲(\frac{۱}{۴})}{۱-\frac{۱}{۱۶}}=\frac{\frac{۱}{۲}}{\frac{۱۵}{۱۶}}=\frac{۸}{۱۵}$$

$$\sin^{۲}\alpha+\cos^{۲}\alpha=۱$$

$$۶۴K^{۲}+۲۲۵K^{۲}=۱\Rightarrow ۲۸۹K^{۲}=۱\Rightarrow K^{۲}=\frac{۱}{۲۸۹}\Rightarrow K=\frac{۱}{۱۷}$$

$$\sin\alpha=\frac{۸}{۱۷}$$

$$\cos\alpha=\frac{۱۵}{۱۷}$$

۱۰ (۲)

$$\frac{\cot\alpha}{\sin\alpha}>۰\Rightarrow\frac{\frac{\cos\alpha}{\sin\alpha}}{\sin\alpha}>۰\Rightarrow\frac{\cos\alpha}{\underbrace{\sin^{۲}\alpha}_{\text{همیشه مثبت}}}>۰\Rightarrow\underline{\cos\alpha>۰}$$

$$۲\sin\alpha<\sin ۲\alpha\xrightarrow{\times\cos\alpha}\underbrace{۲\sin\alpha\cos\alpha}_{\sin ۲\alpha}<\sin ۲\alpha\cos\alpha\Rightarrow\sin ۲\alpha<\sin ۲\alpha\cos\alpha\Rightarrow\sin ۲\alpha(\underbrace{۱-\cos\alpha}_{+})<۰$$

$$\Rightarrow\sin ۲\alpha<۰\Rightarrow ۲\sin\alpha\underbrace{\cos\alpha}_{+}<۰\Rightarrow\underline{\sin\alpha<۰}$$

α در ناحیه چهارم قرار دارد ✓