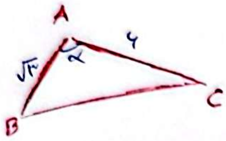
نام و نام خانوادگی ...سهیل بهرامی... پاسخنامه تشریحی تکلیف شماره ...۲۷... کلاس ...یازدهم تجربی B...

۱

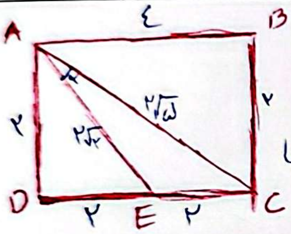


$S=\frac12 AB\times AC\times\sin\hat A=\frac12\times\sqrt3\times6\times\sin\hat\alpha=4.5$

$3\sqrt3\sin\hat\alpha=4.5\Rightarrow\sin\hat\alpha=\frac{\sqrt3}{2}\Rightarrow\alpha=120^\circ$ یا $\alpha=60^\circ$

$\frac{120}{60}=2$

۲



$AE^2=AD^2+DE^2=4+4=8\Rightarrow AE=\sqrt8=2\sqrt2$

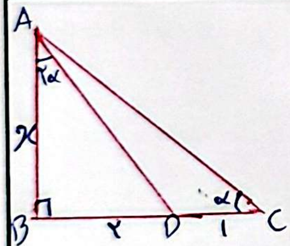
$AC^2=AB^2+BC^2=16+4=20\Rightarrow AC=\sqrt{20}=2\sqrt5$

$\xrightarrow[E\hat AC]{\text{قضیه cosها}}EC^2=AE^2+AC^2-2AE\times AC\times\cos\alpha\Rightarrow4=8+20-2\times2\sqrt2\times2\sqrt5\times\cos\alpha$

$\Rightarrow8\sqrt{10}\cos\alpha=24\Rightarrow\cos\alpha=\frac{3}{\sqrt{10}}\Rightarrow\sin\alpha=\frac{1}{\sqrt{10}}$ $\quad\sin^2\alpha+\cos^2\alpha=1$

$\cot\alpha=\frac{\cos\alpha}{\sin\alpha}=\frac{\frac{3}{\sqrt{10}}}{\frac{1}{\sqrt{10}}}=\frac31=3$

۳



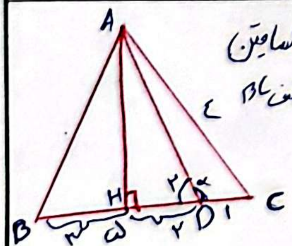
$\xrightarrow[A\hat BC]{\hat B=90^\circ}\cot\alpha=\frac{BC}{AB}=\frac3x\Rightarrow\tan\alpha=\frac{1}{\cot\alpha}=\frac x3$

$\xrightarrow[A\hat DB]{\hat B=90^\circ}\cot2\alpha=\frac{AB}{BD}=\frac x2\Rightarrow\tan2\alpha=\frac{1}{\cot2\alpha}=\frac2x$

$\tan2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}=\frac{\frac{2x}{3}}{1-\frac{x^2}{9}}=\frac{\frac23x}{\frac{9-x^2}{9}}=\frac2x\Rightarrow\frac{6x}{9-x^2}=\frac2x$

$\Rightarrow3x^2=9-x^2\Rightarrow4x^2=9\Rightarrow x^2=\frac94\Rightarrow x=\frac32$ $\qquad\cot\alpha=\frac3x=\frac{3}{1.5}=2$

۴



AH در متساوی الساقین ارتفاع، میانه و عمود منصف BC $\Rightarrow BH=CH$

$\left.\begin{array}{l}BC=BD+DC=6\\BC=BH+CH=2CH\end{array}\right\}\Rightarrow2CH=6\Rightarrow CH=3$

$CD+DH=3\Rightarrow DH=2$

$AB^2=AH^2+BH^2\Rightarrow16=9+AH^2\Rightarrow AH^2=7\Rightarrow AH=\sqrt7$

$\tan\hat D_2=\frac{AH}{HD}=\frac{\sqrt7}{2}\Rightarrow\tan\alpha=-\frac{\sqrt7}{2}$

۵

$2\sin^2x+\cos^2x=\frac43$ $\qquad\sin^2x+\cos^2x=1\Rightarrow\sin^2x=\frac13$ $\qquad\cos^2x=\frac23$

$\tan^2x=\frac{\sin^2x}{\cos^2x}=\frac{\frac13}{\frac23}=\frac12$

۶

$$\frac{\sin^4\alpha+4\cos^2\alpha}{1+\cos^2\alpha} = \frac{(\sin^2\alpha)^2+4\cos^2\alpha}{1+\cos^2\alpha} = \frac{(1-\cos^2\alpha)^2+4\cos^2\alpha}{1+\cos^2\alpha} = \frac{\cos^4\alpha-2\cos^2\alpha+1+4\cos^2\alpha}{1+\cos^2\alpha}$$

$$= \frac{\cos^4\alpha+2\cos^2\alpha+1}{1+\cos^2\alpha} = \frac{(\cos^2\alpha+1)^2}{\cos^2\alpha+1} = \cos^2\alpha+1$$

$$\frac{\cos^4\alpha+4\sin^2\alpha}{1+\sin^2\alpha} = \frac{(1-\sin^2\alpha)^2+4\sin^2\alpha}{1+\sin^2\alpha}$$

$$= \frac{\sin^4\alpha-2\sin^2\alpha+1+4\sin^2\alpha}{1+\sin^2\alpha} = \frac{\sin^4\alpha+2\sin^2\alpha+1}{1+\sin^2\alpha} = \frac{(1+\sin^2\alpha)^2}{1+\sin^2\alpha} = \sin^2\alpha+1$$

$$\cos^2\alpha+1-\sin^2\alpha-1 = \cos^2\alpha-\sin^2\alpha$$

$$\cos^2\alpha-\sin^2\alpha = \cos 2\alpha$$

۷

$$\tan\alpha = \frac{4}{3} = \frac{\sin\alpha}{\cos\alpha} \qquad \cot\alpha = \frac{1}{\tan\alpha} = \frac{3}{4} \qquad \sin\alpha\cos\alpha = \frac{1}{\tan\alpha+\cot\alpha} = \frac{1}{\frac{3}{4}+\frac{4}{3}} = \frac{12}{25}$$

$$\underbrace{\sin\left(\frac{9\pi}{2}+\alpha\right)}_{\cos\alpha}\underbrace{\cos\left(\frac{7\pi}{2}-\alpha\right)}_{-\sin\alpha} - \underbrace{\tan\left(\alpha-\frac{3\pi}{2}\right)}_{-\cot\alpha} = -\sin\alpha\cos\alpha+\cot\alpha = \frac{-12}{25}+\frac{3}{4} = \frac{27}{100}$$

۸

$$3\cos 4x + \underbrace{\sqrt{2}(\sin x-\cos x)}_{2\sin(x-\frac{\pi}{4})} = 3\cos 4x + 2\sin\left(x-\frac{\pi}{4}\right) \overset{x=\frac{\pi}{12}}{=} 3\cos\frac{\pi}{3} + 2\sin\left(\frac{-\pi}{6}\right)$$

$$= 3\cos\frac{\pi}{3} + 2\sin\left(-\frac{\pi}{6}\right) = 3\left(\frac{1}{2}\right) + 2\left(-\frac{1}{2}\right) = 1.5-1 = \boxed{0.5 = \frac{1}{2}}$$

۹

$$\frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha} = \frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}} = \frac{\frac{8\times17-8\times15}{15\times17}}{\frac{-7}{17}} = \frac{16}{15\times-7}$$

$$\frac{16}{15\times-7} = \frac{-16}{105}$$

$$\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} = \frac{2(\frac{1}{4})}{1-\frac{1}{16}} = \frac{\frac{1}{2}}{\frac{15}{16}} = \frac{8}{15}$$

$$\sin^2\alpha+\cos^2\alpha = 1$$

$$64K^2+225K^2 = 1 \Rightarrow 289K^2 = 1 \Rightarrow K^2 = \frac{1}{289} \Rightarrow K = \frac{1}{17}$$

$$\sin\alpha = \frac{8}{17} \qquad \cos\alpha = \frac{15}{17}$$

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$$\frac{\cot\alpha}{\sin\alpha} > 0 \Rightarrow \frac{\frac{\cos\alpha}{\sin\alpha}}{\sin\alpha} > 0 \Rightarrow \frac{\cos\alpha}{\underbrace{\sin^2\alpha}_{\text{همیشه مثبت}}} > 0 \Rightarrow \cos\alpha > 0$$

$$2\sin\alpha < \sin 2\alpha \xrightarrow{\times\cos\alpha} \underbrace{2\sin\alpha\cos\alpha}_{\sin 2\alpha} < \sin 2\alpha\cos\alpha \Rightarrow \sin 2\alpha < \sin 2\alpha\cos\alpha \Rightarrow \sin 2\alpha(\underbrace{1-\cos\alpha}_{+}) < 0$$

$$\Rightarrow \sin 2\alpha < 0 \Rightarrow 2\sin\alpha\underbrace{\cos\alpha}_{+} < 0 \Rightarrow \sin\alpha < 0$$

α در ناحیه چهارم قرار دارد