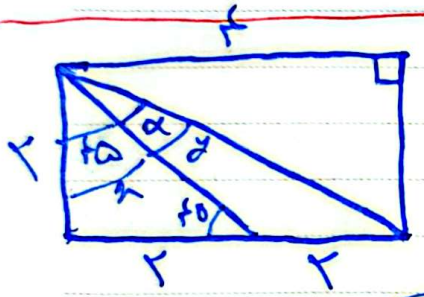
فرهام فروزنده یازدهم پیرو B تکلیف شماره ۲۷

سوال 1

$\frac{1}{2} \times 6 \times \sqrt{3} \times \sin\alpha = 4,5 \Rightarrow \sin\alpha = \frac{9}{9\sqrt{3}}$

$\Rightarrow \sin\alpha = \frac{\sqrt{3}}{2} \begin{cases} \alpha = \frac{\pi}{3} \\ \alpha = \frac{2\pi}{3} \end{cases} \Rightarrow \frac{\alpha_{max}}{\alpha_{min}} = \frac{\frac{2\pi}{3}}{\frac{\pi}{3}} = \boxed{2}$

سوال 2:

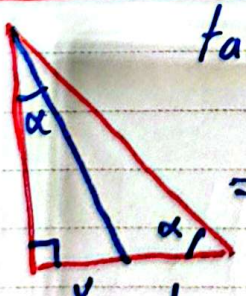


$\tan\alpha = \tan(\beta - \alpha)$

$\tan\alpha = \frac{2}{4} = 1 \Rightarrow \tan\beta = \frac{4}{2} = 3$

$\Rightarrow \tan(\beta - \alpha) = \frac{\tan\beta - \tan\alpha}{1 + \tan\alpha\tan\beta} = \frac{2-1}{1+2} = \frac{1}{3} \Rightarrow \boxed{\cot\alpha = 3}$

سوال 3:



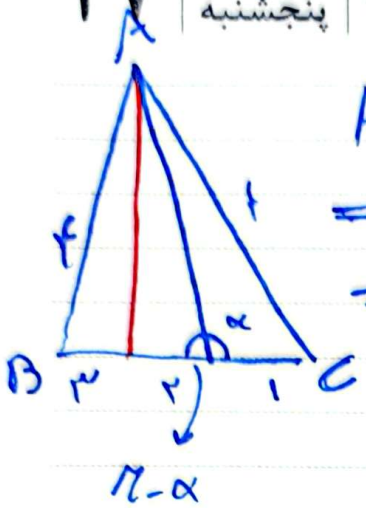
$\tan 2\alpha = \tan(\alpha + \alpha) = \frac{\tan\alpha + \tan\alpha}{1 - \tan^2\alpha}$

$\Rightarrow \tan\alpha = \frac{a}{2}, \tan 2\alpha = \frac{2}{a}$

$\Rightarrow \frac{\frac{2a}{c}}{1 - \frac{a^2}{9}} = \frac{\frac{2a}{c}}{\frac{9-a^2}{9}} = \frac{6a}{9-a^2} = \frac{2}{a}$

$\Rightarrow \tan\alpha = \frac{2\sqrt{2}}{3} = \frac{1}{2} \Rightarrow \cot\alpha = \boxed{2}$

سوال ۴:



$AB^2 = AH^2 + BH^2$

$\Rightarrow 16 = AH^2 + 9$

$\Rightarrow AH = \sqrt{7}$

$\tan(\pi-\alpha) = \frac{\sqrt{7}}{2} \Rightarrow -\tan\alpha = \frac{\sqrt{7}}{2}$

$\Rightarrow \tan\alpha = -\frac{\sqrt{7}}{2}$

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سوال 5:

$2\sin^2\alpha + \cos^2\alpha = \frac{4}{3} \Rightarrow \sin^2\alpha + 1 = \frac{4}{3}$

$\Rightarrow \sin^2\alpha = \frac{1}{3} \Rightarrow \sin\alpha = \frac{1}{\sqrt{3}}$

$\tan^2\alpha = \frac{\sin^2\alpha}{\cos^2\alpha} = \frac{\sin^2\alpha}{1-\sin^2\alpha} = \frac{\frac{1}{3}}{\frac{2}{3}} = \boxed{\frac{1}{2}}$

سوال 6:

$$\frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha}$$

$\cos^2\alpha = 1-\sin^2\alpha$ $\sin^2\alpha = 1-\cos^2\alpha$

$$\Rightarrow \frac{\sin^4\alpha + 4(1-\sin^2\alpha)}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4(1-\cos^2\alpha)}{1+\sin^2\alpha} = \frac{(2-\sin^2\alpha)^2}{2-\sin^2\alpha} - \frac{(2-\cos^2\alpha)^2}{2-\cos^2\alpha}$$

$$\Rightarrow (2-\sin^2\alpha) - (2-\cos^2\alpha) = \cos^2\alpha - \sin^2\alpha = \cos(2\alpha)$$

سوال ۷:

$$\sin\left(\frac{9\pi}{2}-\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right) - \tan\left(\alpha-\frac{3\pi}{2}\right)$$

$$\cos\alpha - \sin\alpha + \cot\alpha \xrightarrow[\text{ربع سوم}]{\tan\alpha=\frac{4}{3}} \cos\alpha = -\frac{3}{5},\ \sin\alpha = -\frac{4}{5}$$

$$\cot\alpha = \frac{3}{4}$$

$$\Rightarrow \left(-\frac{3}{5}\right) - \left(-\left(-\frac{4}{5}\right)\right) + \frac{3}{4} = -\frac{12}{25} + \frac{3}{4} = \frac{27}{100} = 0/27$$

سوال ۸

$$2\cos^2\frac{\pi}{8} + \sqrt{2}\sin\frac{\pi}{12} - \sqrt{2}\cos\frac{\pi}{12}$$

$$\Rightarrow \frac{3}{2} + \sqrt{2}\left(\sin\frac{\pi}{12} - \cos\frac{\pi}{12}\right) \Rightarrow \sin\frac{\pi}{12} < \cos\frac{\pi}{12}$$

$\hookrightarrow A < 0$

$$\Rightarrow A^2 = \left(\sin\frac{\pi}{12} - \cos\frac{\pi}{12}\right)^2 \Rightarrow A^2 = 1 - \sin\frac{\pi}{6}$$

$$\Rightarrow A = -\frac{1}{\sqrt{2}} \Rightarrow \frac{3}{2} + \sqrt{2}\left(-\frac{1}{\sqrt{2}}\right) = \boxed{\frac{1}{2}}$$

سوال ۹ :

$\tan\left(\frac{\alpha}{2}\right)=\frac{1}{4} \rightarrow \tan\alpha=\frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}}=\frac{8}{15}$

$\hookrightarrow \sin\alpha=\frac{8}{17}, \cos\alpha=\frac{15}{17}$

$$\frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha}=\frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}}=-\frac{16}{105}$$

سوال ۱۰

$2\sin\alpha<\sin 2\alpha \Rightarrow 2\sin\alpha<2\sin\alpha\cos\alpha$

$\hookrightarrow 2\sin\alpha-2\sin\alpha\cos\alpha<0$

$\Rightarrow 2\sin(1-\cos\alpha)<0$

$\Rightarrow \sin\alpha<0$ ← ربع ۳ و ۴

$\Rightarrow \frac{\cos\alpha}{\sin\alpha}>0$ ← هم‌علامت ← ربع ۳